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Nonlinear Contracts, Zero Profits and Moral Hazard

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Contracts that base payments on an ex post variable are examined. It is shown that a quadratic contract form may elicit truthful responses from auction participants and offer zero expected profits to the winning bidder, but not eliminate adverse incentives ex post. A general impossibility theorem is proven. This theorem establishes that, regardless of functional form, no contract can offer zero expected profits and resolve the ex post moral hazard problem.

The problem of how to sell a single unit of a good to buyers with valuations not known to the seller has been studied in recent years. An auction of the good in question has been the primary approach to this problem. Many properties of real-world auctions are developed in Milgrom and Weber (1982), including the Revenue Equivalence Theorem, which states that the most commonly used auctions (e.g. sealed bid and oral ascending) all produce the same expected revenue to the seller. Any one of these auctions is optimal for the seller when the buyers' valuation is private information and these values are independently distributed (cf. Riley and Samuelson, 1981; Myerson, 1981; and Maskin and Riley, 1984). This result does not hold when buyers are risk-averse (Matthews, 1983). Even in a risk-neutral environment, however, the seller can only expect to receive the second-highest valuation of the good. The asymmetry of information appears to be costly to the seller, for if he knew everyone's private valuation, he could sell the good on a take-it-or-leave-it basis to the buyer with the highest valuation.

Very little attention has been paid to auctions in which the winner's private valuation, or some proxy for the winner's private valuation, will eventually be known. The actual value to the winner, however, becomes known in many extant contracts. For example, the amount of oil recovered from offshore oil tracts and the costs of recovery are frequently revealed, and the actual cost of government procurement contracts is generally established after auditing. Thus it is feasible, in many instances, to condition payment on the outcome, as well as the bid. This was recognized by McCall (1970), Baron (1972), Holt (1979), Weitzman (1980) and Samuelson (1983) for military procurement contracts. Their attention, however, was confined to linear contracts.[1] The purpose of this paper is to investigate feasible allocations from the class of all contract forms when the contract may depend on the buyer's valuation and the bid.

In contrast to the extant auction literature, it is feasible, as we show in Theorem 1, to extract all of the rents from the buyer. That is, it is possible to sell a good to the buyer with the highest valuation at his valuation. However, the contract we develop to prove this result creates a perverse incentive. After the selection is complete, the winning buyer would be better off with a lower valuation. If the winning buyer can influence his valuation after selection (i.e.

if a moral hazard situation exists), then he would be expected to do so. We show that this perverse incentive exists in all contracts that extract the value of private information from the winning bidder. Thus, there is a trade-off between providing proper incentives and extracting rents.

The analysis in this paper is presented in terms of a buyer or principal tendering a contract, rather than the usual auction of a good, because government procurement provides the major application for our results. Thus, instead of seeking the highest price in the auction, the principal desires to purchase the good at the lowest price. In addition, the commonly cited independent private values model is assumed to represent the contracting environment.[2]

I. The Model

There are N risk-neutral, non-cooperative, profit-maximizing firms submitting sealed bids for the right to produce a pre-specified project. The cost of producing this project is summarized by

$$(1) \qquad C_i = C_i^* + \varepsilon \qquad (i = 1, 2, \ldots, N)$$

where ε is a continuous random variable with zero mean and a density function, $f(\cdot)$, common to all firms. The interpretation of ε is that it subsumes all those cost components that were not completely anticipated during bidding. For example, the project may be delayed because of particularly bad weather conditions, which might increase the cost of production above what had been anticipated. The anticipated or expected cost of the project to the ith firm is given by C_i^*. To account for the fact that firms are likely to have differing expected costs, this component of project costs is assumed to vary across firms following a distribution denoted by $G(\cdot)$ with a density $g(\cdot)$.

During bidding, a firm knows its expected costs but not those of any other firm. The principal, who is awarding the contract, does not know the expected cost of any firm. If the principal could directly observe this information, then there would be no difficulty in minimizing procurement costs because he would simply offer the project to the firm with the lowest expected costs at a fixed price equal to expected costs. However, because of the asymmetry of information, the principal must discover the relative efficiency of these firms by an indirect approach. A sealed-bid auction is used for this purpose, although our results are not generally tied to this type of auction. The firm with the lowest bid wins the auction.

The principal offers a contract of the form $p(b, C)$, where the winning firm with bid b and cost C is paid $p(b, C)$. This function, or contract, might represent any of the commonly used procurement contracts. For example, it might represent a fixed-price contract, $p(b, C) = b$; a cost-plus contract, $p(b, C) = C$; a cost-plus-incentive contract, $p(b, C) = C + \beta(b - C)$, which includes the fixed-price ($\beta = 1$) and cost-plus ($\beta = 0$) as special cases, or nonlinear contract forms.

We examine symmetric Nash bidding strategies $B(C_i^*)$ where, if all other firms use this bidding strategy, bidding $B(C_i^*)$ is optimal for the remaining firms.[3] Given that all firms except one use the strategy $B(C_i^*)$, the remaining firm will win the contract with probability $[1 - G\{B^{-1}(b)\}]^{N-1}$. Thus, this firm chooses the bid b that maximizes

$$E\Pi(b, C_i^*) = \int \{p(b, C_i^* + \varepsilon) - (C_i^* + \varepsilon)\} f(\varepsilon)\, d\varepsilon [1 - G\{B^{-1}(b)\}]^{N-1}. \quad (2)$$

An optimal solution is described in the following result.

Theorem 1. The contract

$$p(b, C) = C - (b - C)^2 + \sigma^2,$$

where σ^2 is the variance of ε, induces honesty in bidding, selects the lowest cost firm, and provides zero profits for all firms.

Proof. First, note that

$$E\Pi(b, C_i^*) = \left\{\sigma^2 - \int (b - C_i^* - \varepsilon)^2 f(\varepsilon)\, d\varepsilon\right\}[1 - G\{B^{-1}(b)\}]^{N-1}$$
$$= -(b - C_i^*)^2 [1 - G\{B^{-1}(b)\}]^{N-1}.$$

Thus, $B(C^*) = C^*$. Because each firm reveals its costs, the efficient firm is selected. Finally, zero profits is obvious. Q.E.D.

Theorem 1 establishes that it is possible to obtain the lowest price from an auction using a quadratic contract when the bidders' private costs are uncertain. This solution is not available when a linear contract is specified. Differentiating (2) and rearranging terms reveals that $E\{p(b, C)\} > C^*$ for all linear contracts that satisfy the second-order conditions as a strict inequality. Therefore, fixed-price or cost-plus-incentive contracts offer positive economic profits to bidding firms.[4]

This contract appears to have only one undesirable property: after the fact, or *ex post*, profit increases in cost C whenever $C < C^*$. If there is a cost underrun, the firm has an incentive to increase or pad its costs. This perverse incentive creates a moral hazard situation for the principal, which arises whenever $p_2(b, C) > 1$. In this case, *ex post* profits increase as C increases. The remedy for this problem is to find a contract that reverses this inequality. Unfortunately, such a contract is inconsistent with zero expected profits, as the following result shows.

Theorem 2. If firms maximize expected profits using a symmetric Nash bidding strategy, then there is no contract that will jointly satisfy $E\{p(b, C)\} = C^*$ and $p_2(b, C) < 1$ in equilibrium.

Proof. In equilibrium $b = B(C^*)$. Thus,

$$\frac{\partial E\{p(b, C)\}}{\partial C^*} = \int \{p_1(b, C) B'(C^*) + p_2(b, C)\} f(\varepsilon)\, d\varepsilon.$$

The condition $p_2(b, C) < 1$ implies

$$\frac{\partial E\{p(b, C)\}}{\partial C^*} < 1 + B'(C^*) E\{p_1(b, C)\}$$

and the zero profits condition, $E\{p(b, C)\} = C^*$, implies

$$\frac{\partial E\{p(b, C)\}}{\partial C^*} = 1$$

which, from the maximization of (2), can occur only if $Ep_1(b, C) = 0$; and thus a contradiction arises. Q.E.D.

The most important implication of this theorem is that the principal always faces a trade-off between resolving *ex post* incentive problems and minimizing costs in the bidding or *ex ante* phase of contracting. In other words, within the confines of this model, the joint problems of efficient selection and moral hazard cannot be completely resolved when there is asymmetric information. The principal's lack of information about expected costs implies that a full-information, cost-minimizing contract is not obtainable. As such, Theorem 2 shows that the contract selected will provide the winning firm with the possibility for positive profits either built into the payment function (*ex ante*) or as a result of cheating (*ex post*). In general, of course, the principal may select a contract that offers positive profits both *ex ante* and *ex post.*

II. Final Remarks

The significance of this analysis is twofold. First, providing incentives to minimize costs when there is asymmetric information is costly, even if it is possible to condition the contract on an *ex post* variable. Second, contrary to the auctions where payment is conditioned only on the bid, it is possible to extract all the rents from the bidders and still select the most efficient firm, but only at the cost of creating perverse incentives *ex post.*

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NOTES

1. While these papers all investigate linear contracts, they differ from each other significantly. McCall (1970) analyses whether linear incentive contracts allow the government to distinguish between high- and low-cost bidders; Baron (1972) studies the effects of taxes and risk aversion on bids; and Holt (1979) analyses bidding behaviour and procurement costs when bids are determined in a symmetric Nash equilibrium model. Weitzman (1980) and Samuelson (1983) consider the problem of moral hazard in designing efficient contracts; Samuelson uses risk aversion in a bidding model to generate a trade-off between efficient selection and moral hazard, and Weitzman uses risk aversion to design an efficient linear contract after a firm has been selected. In contrast, this analysis develops such a trade-off without assuming risk-averse bidders.
2. The terminology is attributed to Milgrom and Weber (1982).
3. It follows immediately that honesty can be induced merely by offering the payment function $\hat{p}(b, C) = p\{B(b), C\}$, with the same distribution of winners, costs and payments. This is a corollary of the Revelation Principle.
4. The cost-plus contract is the only linear contract that may offer zero profits to the bidding firms. This contract, however, does not guarantee that the most efficient firm is selected. In the cost-plus contract, expected profits are zero for any bid. Thus, any bidding strategy may be used. Consequently, it cannot be claimed that the cost-plus contract selects the lowest-cost bidder, as this solution is one of many possible equilibria.

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