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By R. Preston McAfee and John McMillan*

We characterize coordinated bidding strategies in two cases: a weak cartel, in which the bidders cannot make side-payments; and a strong cartel, in which the cartel members can exclude new entrants and can make transfer payments. The weak cartel can do no better than have its members submit identical bids. The strong cartel in effect reauctions the good among the cartel members. (*JEL* D44, D82, L41)

A successful cartel must overcome at least four obstacles. First, the conspirators must devise some mechanism for dividing the spoils. Each cartel member has an incentive to argue for a bigger share. Second, an agreement is worthless without some way of enforcing it. Since contracts to fix prices cannot usually be written, any collusive agreement must be designed to be self-enforcing. Third, collusion contains the seeds of its own destruction. The high profits earned in a successfully colluding industry attract new firms into the industry; the competition from those new entrants then tends to destroy the collusive arrangements. Fourth, the victims of the cartel, on the other side of the market, may take actions to destabilize it. The first of these problems is empirically at least as important as the other three: for example, in a sample of international cartels that were temporarily successful but then broke down, almost half were destroyed by internal squabbling over how to share the profits (Paul Eckbo, 1976 Ch. 3). Most of the U.S. Department of Justice's bid-rigging convictions begin when one of the cartel members, dissatisfied with his share of the spoils, turns in his coconspirators.

The main subject of this paper is how cartels overcome the division-of-the-spoils difficulties, in the specific context of bidding at auctions.[1] The colluding bidders must overcome an adverse-selection problem: they do not know how much each of their fellow cartel members is willing to pay for the item being sold. We shall derive the optimal mechanism for the cartel to use to decide who receives the item and how the proceeds are distributed. Our model will also have something to say about two of the other cartel problems listed above: entry deterrence and active seller responses. We shall, however, have nothing to add to what has already been said about cartel enforcement (see e.g., George Stigler, 1964; Dilip Abreu et al., 1986).

We examine primarily all-inclusive bidder cartels at sealed-bid first-price auctions,[2] except in Section VI, where we offer a partial analysis for bidder cartels that contain

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[1] Biddings conspiracies are prevalent enough to have added some exotic locutions to the English language. Cartels are variously called "rings," "pies," and "kippers." A "schlepper" is an insincere bidder attracted solely by the cartel's profits, and a "shill" is a phony bidder used by the auctioneer to drive up the price. A "knockout" is a private auction held by the cartel to determine which member gets the item and how much he pays the other members.

[2] On collusions not involving all bidders and on collusion in English and second-price auctions, see Daniel Graham and Robert Marshall (1987), Thomas von Ungern-Sternberg (1988), and George Malaith and Peter Zemsky (1991).

only some of the bidders. Our analysis will explain two commonly observed forms of cartel organization. One set of results is about weak cartels, by which we mean cartels whose members are unable to make transfer payments among themselves. We will show that in weak cartels all bidders submit exactly the same bid. This may explain why, in the bidding for government contracts, it has been often the case that all bids are identical to the last cent. The second set of results is about strong cartels: cartels that can both make transfer payments and exclude new entrants. A common method by which a cartel decides which bidder is to get the item and the size of the transfers is to hold its own illicit auction. Our model will show that this is an optimal mechanism for the cartel.

While explicit collusion is illegal in the United States, the legality of implicit collusion is ambiguous; lawyers are still discussing the nature of the evidence that is needed in order to prosecute for collusion. Even if it is illegal, implicit collusion is more difficult to detect and prove than explicit collusion. Thus, although we shall find that a cartel that makes transfer payments among its members is more profitable than one that does not, bidders may prefer to collude implicitly if the risk of detection and the severity of punishment of explicit collusion (not modeled here) are sufficiently great.

In terms of the amounts of money involved, some of the most significant cases of collusion occur in the bidding for government contracts.[3] Although our model will be expressed in terms of selling, with appropriate sign changes it can easily be converted into a model of contract bidding.

I. Enforcement

In order to collude, the bidders must resolve their asymmetric-information problem: they must have some way of selecting a winner and a winning bid. We model this by supposing that the bidders use a *mechanism* prior to the realization of the information, that is, a decision rule that assigns bids and (in some cases) transfers, based on the bidders' reports. The phases-of-the-moon system used by the electrical-equipment conspiracy (Richard Smith, 1961) is an example of a mechanism. By the revelation principle (Roger Myerson, 1985), we may without loss of generality restrict attention to direct, incentive-compatible mechanisms, in which each bidder reports his valuation to the mechanism and has incentives to do so honestly. The revelation principle states that the outcome of any mechanism that is not incentive-compatible can be mimicked by one that is incentive-compatible, so that honesty can be assumed without loss of generality. A direct mechanism, then, takes the vector of the bidders' reports of valuations and dictates bids and (perhaps) transfers to each bidder. We shall assume that the cartel designs the mechanism to maximize the *ex ante* (before the valuations are known) sum of bidders' expected profits in the auction.

The seller's behavior is passive. The seller announces a reserve price and sells at the highest bid to the highest bidder. In the event of a tie, the seller is presumed to randomize equally over the bidders with the highest bid. It will transpire that such behavior on the part of the seller, which is employed in most government procurement auctions, is not a best response against the existence of a cartel. Therefore, we are assuming that the seller either does not know he faces a cartel or is bound to the rules of the sealed-bid auction by law.

The model we develop is static: it focuses on behavior in a single auction. However, the cartel rules require an enforcement mechanism, for there will be an individual incentive to defect. Although we shall consider various kinds of cartel mechanisms, it will turn out that the mechanism works by assigning either a maximum amount that any member may bid or (in the case in which side-payments are possible) by designating the identity of the winner and the

[3]According to the U.S. Justice Department's antitrust chief, bid-rigging by highway contractors increased the cost of building roads by 10 percent or more (*New York Times*, 4 April 1988, p. 29). Bid-rigging is widespread in Japan's public-works contracting and has contributed to U.S.–Japan trade friction (McMillan, 1991).

designated winner's bid. In either case, the information necessary to implement an enforcement strategy (i.e., to detect deviations from actions the mechanism prescribes) is at most the identity and bid of the winning bidder. We shall assume that these two pieces of information become public. This information is made available, by law, in government procurement auctions (with the exception of certain Department of Defense projects) (see Stigler, 1964; George Hay and Daniel Kelley, 1974; Richard Posner, 1976 p. 62; Charles Geiss and John Kuhlman, 1978).

The enforcement needed to ensure that the members comply with the cartel mechanism can come from one of two sources. First, the cartel may hire an enforcer who punishes any observed deviating bidders—an organized-crime approach. The alternative avenue is to appeal to a grim trigger strategy in an infinitely repeated auction context[4] (Abreu et al., 1986). Although we do not model dynamics explicitly, it is clear that cooperation is one of the equilibria in the infinitely repeated noncooperative game. A deviating bidder can be threatened with noncooperative profit levels in all future auctions should he win the current auction when the mechanism dictated otherwise. This threat will be sufficient to deter deviations if discounting is sufficiently low. Although maximal profits represent only one equilibrium among a plethora of repeated-game equilibria (James Friedman, 1986), a cartel that is choosing its mechanism has an obvious incentive to coordinate on this equilibrium. Anecdotal evidence from antiques and artwork auctions indicates that retaliatory strategies are in fact used to enforce collusion: "When one of the members of the ring goes against his partners..., or the ring falls out for one reason or another, then it works very much to the seller's advantage as vindicative competition leads to crazy prices" (Jeremy Cooper, 1977 pp. 37–8). Similarly, retaliation in subsequent auctions was the enforcement mechanism used by the electrical-equipment conspiracy (Smith, 1961 p. 175) and by highway-construction cartels (Steven Flax, 1983 p. 80).

We assume, therefore, that some punishment is available to the cartel, so that no cartel member will ever disregard the mechanism when the mechanism dictates that he bid to lose. Because our primary interest is in examining the constraints on the cartel that result from the privacy of the cartel members' information, the model to be developed endows the cartel with the ability to ensure obedience to the cartel mechanism's orders, but not with the ability to prevent any cartel member from lying to the cartel mechanism, except insofar as the cartel mechanism can be designed so as to make honesty optimal.

II. Modeling Issues

We suppose that a unique item is to be sold by sealed-bid auction to one of a set of risk-neutral bidders. The distinctive feature of an auction is asymmetric information: if the seller knew the bidders' demands, he would simply post a price. We model the asymmetry of information by assuming that bidder i, $i=1,\ldots,n$, knows his own willingness to pay, v_i, while all the other bidders and the seller perceive v_i as an independent draw from a cumulative distribution F. Assume that F has a differentiable density f with support $[0, v_h]$.

The assumption of independent draws, which is ubiquitous in the mechanism-design literature, rules out correlated valuations and therefore may make the model inapplicable to some real-world auctions in which collusion occurs. For example, if the perceived value of a piece of art depends on its unknown future value and if the bidders' predictions of the future value are correlated, then this model does not apply (Paul Milgrom and Robert Weber, 1982). However, the independent-private-values assumption is not inconsistent with there being some factors that influence all bidders' valuations. For example, if the determinants

[4]In order to appeal to the repeated-auction interpretation, we must assume that the bidders draw their valuations anew before each auction, so that it is the same private-information game that is repeated. For an alternative case, see Peter C. Cramton and Thomas R. Palfrey (1990).

of the artwork's future price are known and equally weighted by all of the bidders, the independence assumption is satisfied. Also, if the actual value is $v_i = p + w_i$, where w_i represents the bidder's personal valuation of the item and p is its unknown future market price, the same for all bidders and independent of the w_i's, then the independent-private-values model still applies. Thus, the analysis to follow applies to situations with common-value elements, provided all the bidders have the same information about the common aspects.

A further reason for focusing on the independent-private-values case is that, in the pure common-value case, the optimal cartel mechanism is simple if the cartel members can communicate with each other. In the pure common-value case, efficiency is attained regardless of which bidder wins. The cartel can therefore use some exogenous method to pick which of its members is to win. It then asks each bidder to report his private information about the item's true value. Since the report does not affect the probability of winning, there is no incentive to misrepresent this information. Based on the pooled information, the cartel decides whether the expected value of the item exceeds the reserve price; if it does, the assigned bidder bids the reserve price without competition (cf. McAfee et al., 1989).

The cartel mechanism works as follows. After the bidders have reported their valuations to the mechanism, the ith bidder is awarded the good with probability $h_i(v_i, \mathbf{v}_{-i})$, where $\mathbf{v}_{-i}$ represents the vector of others' reports (equal to true values in equilibrium). Bidder i's expected profit, if he has value v_i but reports w_i, is

$$(1) \quad \pi_i = E_{-i}\{v_i h_i(w_i, \mathbf{v}_{-i}) - T_i(w_i, \mathbf{v}_{-i})\}$$

where E_{-i} is the expectation over $\mathbf{v}_{-i}$ and T_i is the expected payment by bidder i. If transfers are prohibited, T_i is the bid weighted by the probability of winning. If transfers are allowed, T_i is the weighted bid plus the net transfer. From Roger Guesnerie and Jean-Jacques Laffont (1984), incentive compatibility is equivalent to[5]

$$(2) \quad \frac{d\pi_i}{dv_i} = E_{-i} h_i(v_i, \mathbf{v}_{-i})$$

and

$$(3) \quad \frac{\partial}{\partial v_i} E_{-i} h_i(v_i, \mathbf{v}_{-i}) \geq 0.$$

On a priori grounds, one can imagine four kinds of collusive mechanism, defined by the instruments available. Listed roughly in order of ease of detection by outsiders, these are: first, the *tacit* mechanism, in which there are no transfers and the ith bidder's bid depends only on his own valuation; second, the *coordinative* mechanism, in which there are no transfers, but the bids may depend upon the entire vector of reports; third, the *transfer* mechanism, which has side-payments, but they must sum to zero for every realization of the vector of valuations; and fourth, the *budget-breaking* mechanism, in which the side-payment constraint is relaxed so that transfers need sum to zero only on average. Tacit collusion is probably legal in the United States and is in any case probably undetectable, since no contact among the bidders is needed to operate the mechanism. The bidders need not even know who the other bidders are. A coordinative collusion runs some risk of detection, since the communications might be intercepted. Transfers generate an additional potential source of evidence. Finally, a budget-breaking collusion is still more risky, as it must employ a third party (the budget-breaker) who does not value the item

[5]The proof is as follows. Let $a(w) = E_{-i} h_i(w, \mathbf{v}_{-i})$ and $b(w) = E_{-i} T(w, \mathbf{v}_{-i})$, so $\pi(v, w) = va(w) - b(w)$. Note that $0 = \pi_w(v, v) = va'(v) - b'(v)$ is equivalent to (2). Differentiating $0 = \pi_w(v, v)$ yields $0 = \pi_{vw}(v, v) + \pi_{ww}(v, v)$, and therefore the necessary second-order condition yields $0 \leq \pi_{vw}(v, v)$, which is (3). To obtain sufficiency, note that $\pi_{vw}(w, w) \geq 0$ implies $\pi_{vw}(v, w) \geq 0$ (v has dropped out) and thus $\pi_w(v, w) \gtrless 0$ as $w \gtrless v$, implying $\pi(v, w) \leq \pi(v, v)$. We shall abuse notation and write $\pi(v) = \max_w \pi(v, w) = \pi(v, v)$. This logic was first applied by Myerson (1981).

being sold. We shall investigate all four types of cartel (although, as we shall find, there are actually only two different types, defined by the presence or absence of transfers).[6] The mechanism is chosen to maximize *ex ante* profits prior to the realization of valuations (in contrast to Cramton and Palfrey [1990]).

In the next two sections we shall take as given the seller's policy: the seller offers to sell the item by sealed bid to the highest bidder, provided the bid exceeds some announced level r. This may be a reserve price set at the lowest possible valuation that a bidder could have, at the seller's own valuation of the item, or at some level higher than the seller's valuation so as to exploit the seller's monopoly power. Alternatively (and in some cases more realistically), r might represent the cartel members' shared perception of how low a bid they can get away with, without either causing the seller to refuse to sell or arousing the suspicion of the antitrust authorities. We shall for brevity call r the minimum price, and we assume that it is known to all bidders.

III. Weak Cartels

For the first result of this section, we assume that the cartel operates under a handicap: the cartel members are unable to make transfer payments to each other. We leave this restriction unexplained, but one reason might be that transfers give rise to a risk of prosecution by the antitrust authorities that is sufficient to induce the cartel members to eschew their use.

The cartel mechanism is efficient provided bidder i wins if and only if his value exceeds both the minimum price and all the other bidders' values. The latter occurs with probability $F(v_i)^{n-1}$. Hence efficiency is characterized by

$$(4)\quad E_{-i}h_i(v_i,\mathbf{v}_{-i}) = \begin{cases} F(v_i)^{n-1} & v_i \geq r \\ 0 & v_i < r. \end{cases}$$

A profit-maximizing cartel clearly prefers efficiency in the absence of incentive constraints, so that the pie to be divided among the members is as large as possible. However, we shall find that the impossibility of transfers prevents efficient collusion. The bidder who values the item the most does not necessarily get it: collusion is inherently limited.

LEMMA: *If a cartel member whose value is less than or equal to the minimum price r is constrained to earn zero profit, then efficiency implies noncooperative profit levels.*[7]

Assume that it is common knowledge that, in the event of two or more bidders being tied for the highest bid, the seller awards the good randomly, with equal likelihood to each of the tied bidders.

Let $H(v) = [1 - F(v)]/f(v)$. We consider two cases. First, $H'(v) \geq 0$ for all $v \in (0,\infty)$; and second, $H'(v) < 0$ for all $v \in (0, v_h)$. The latter can be thought of as the more likely case, as it is satisfied by most common distributions. The expected value of $H(v)$ (with expectations taken over the distribution of the highest valuation) is the expected rent earned by the winning bidder in a noncooperative auction (see McAfee and McMillan [1987] for details). Thus, $H(v)$ is an index of the winning bidder's profit in expected terms, and $H'(v) < 0$ implies that high-value bidders produce relatively less profits.

THEOREM 1: *Suppose transfers are impossible. In the case $H'(v) \geq 0$, optimal implicit collusion involves noncooperative bidding. In the case $H'(v) < 0$, it has*

$$(5)\quad B(v_i,\mathbf{v}_{-i}) = \begin{cases} 0 & v_i < r \\ r & v_i \geq r. \end{cases}$$

Thus, if transfers are prohibited, optimal collusion [in the case $H'(v) < 0$] has every

[6] A related problem is collusion in a Cournot oligopoly when the firms' production costs are private information. This has been modeled by Cramton and Palfrey (1990) and Kevin Roberts (1985).

[7] All proofs are in the Appendix.

bidder who values the item at more than the minimum price submitting the same bid, namely, the minimum price. This is because, in the absence of side-payments, incentive compatibility requires that the good be awarded stochastically, with equal probability of being awarded to anyone whose value is larger than the minimum price: any attempt to arrange that the highest-value bidder wins generates incentives for the bidders to misstate their valuations. By submitting equal bids, the bidders in effect use the seller as their randomizing device.

The mechanism in Theorem 1 works by having the bidders report their valuations to the mechanism (correctly, since incentive-compatibility constraints are imposed), and the mechanism then recommends a bid to each bidder (which it is in the bidder's interest to adhere to). The proof of Theorem 1 allowed the mechanism to recommend bids that are a function of all bidders' reports. However, as Theorem 1 shows, this is not needed: each bid is a function of the bidder's own report alone. This means that the optimal mechanism can be implemented in a decentralized way: the bidders need not actually report to the mechanism, but simply bid r if and only if their valuations exceed r. In this sense, although the analysis allows the possibility of coordination, the optimal mechanism can be implemented without coordination.

Bidding according to the phases of the moon is an example of a correlated equilibrium. Can the cartel do better than using identical bidding by going to a correlated equilibrium? The answer is no. This is because our analysis allowed the bidders to communicate with the mechanism. The set of equilibria in such a game includes the set of correlated equilibria: the revelation principle picks out all correlated equilibria (Myerson, 1985).

Collusion without transfers has two effects relative to noncooperative bidding. First, the payment to the seller is reduced. Second, the average valuation of the winning bidder is decreased, because the winner is no longer efficiently selected. The condition $H'(v) < 0$ guarantees that the former effect outweighs the latter.

For $H'(v) \geq 0$ for all v, it must be that the density function $f(v)$ is sufficiently negatively sloped everywhere. While this is obviously unlikely to be satisfied, it is satisfied for the exponential distribution. In this case, the scheme (5) does not improve on noncooperative bidding. Another case for which intuition suggests that identical bidding would break down occurs when the density function is such that, with high probability, a bidder's valuation is either very high or very low. In this case, the monotonicity of $H(v)$, assumed in Theorem 1, is violated.

The theoretically optimal scheme (5) has a striking real-world counterpart. According to F. M. Scherer (1970 p. 182), "Each year the federal and state governments receive thousands of sets of identical bids in the sealed bid competitions they sponsor." The pervasiveness of identical bidding in U.S., Canadian, and European government contracting has been noted also by Vernon Mund (1960), Paul Cook (1963), Hay and Kelley (1974), William Comanor and Mark Schankerman (1976), Organisation for Economic Cooperation and Development (1976 p. 20), and Christopher Green (1985 pp. 176–7).[8] Despite its prevalence, however, the phenomenon has not heretofore been satisfactorily explained. Why do the bidding firms choose such an apparently naive form of coordination? The answer, implied by Theorem 1, is that, given the asymmetry of information—the firms cannot observe each other's production costs—identical bidding is the best the cartel can do short of using side-payments.

We now consider an alternative (but, we shall show, equivalent) limitation on the bidders' ability to collude. This is based on the empirical observation that the most common reason for cartels to founder is their inability to prevent entry. We consider an extreme form of entry possibilities. Suppose, in addition to the n serious bidders, there is a very large number of bidders with

[8]Remarkable precision can be achieved: for example, in one sealed-bid tender to a Canadian local government, all nine bids were for $6,009.15 (Green, 1985 p. 177).

low values, values less than r, and that these low-value bidders cannot be identified directly from observation. If the cartel is open to anyone, these individuals would have an incentive to join, in order to participate in the cartel's sharing of profits, although they would never actually win an item. If there are enough of these individuals, the cartel must discourage them from claiming that they are actually high-value-distribution types. One way to do this is to offer zero profits to bidders with low values:

$$\pi_i(r) = 0. \tag{6}$$

Note that, if these nonserious bidders, or "schleppers," come from distributions which have a positive probability of values in excess of the reserve price, then one of them may be the efficient owner of the item and should be included in the cartel. Because of the asymmetry we have assumed, this environment is not encompassed by the present model. However, if the schleppers are drawn from a distribution with support below r, so that they have no effect on the noncooperative allocation, they may still desire to join the cartel. A cartel that faces sporadic real entry (by bidders who will receive draws from distribution F) may wish to design the mechanism to deter participation of schleppers.

One justification for this is as follows. If there are infinitely many bidders with distribution G, with maximum r, a seller will never post a reserve price less than r, since someone is willing to pay r. If there are n bidders with valuations to be drawn from F (but not yet drawn), any one of these bidders would like to form a cartel that excludes the G-type bidders, because they dissipate the rents. If the distributional type cannot be identified, then $\pi_i(r) \leq 0$ is necessary.[9]

Although this schlepper story is clearly an incomplete description of a cartel entry problem, the next theorem demonstrates the force of the entry-exclusion constraint (6).

THEOREM 2: *If (6) holds, the bidders' expected profits are maximized by the mechanisms of Theorem 1, even if transfers are possible.*

Thus, the no-entry constraint (bidders with value equal to the reserve price earn zero profits) means that side-payments are useless; the optimal mechanism can be implemented without transfers.

Theorems 1 and 2 establish a three-way equivalence among cartels: (i) without transfers and without coordination; (ii) without transfers and with coordination; and (iii) with transfers but with zero profit at the reserve price. The intuition for this is that, first, the inability to make transfers implies that profit at the minimum price must be zero (because, without transfers, a bidder with value less than r who wins earns negative profit, so it is better for the cartel [maximizing total expected profit] for such a bidder never to win). Second, profit at the minimum price being constrained to be zero implies that the maximum attainable cartel profit can be achieved without using transfers [because (6) implies that some bidders receive zero transfers; thus any transfer scheme cannot be lump-sum and the highest-valuation bidder who fails to receive a transfer has an incentive to overstate his valuation].

Sometimes the cartel might choose a coordinated-bidding mechanism instead of the identical-bidding mechanism of Theorem 1. This mechanism would have some device (such as the phases of the moon) to choose which bidder's turn it is to bid. This bidder is then asked if he wants the item at the

[9]In a cartel practice among bidders for government contracts in Europe described by the Organisation for Economic Co-operation and Development (1976 p. 22), the cartel is able to exclude what we describe as nonserious bidders: "No compensation is paid to firms suspected of participating in meetings of would-be tenderers held before the tender takes place simply in order to obtain compensation; ... compensation will be paid only if tenderers are in possession of a 'claim': a tenderer obtains a claim after completing a building, for example. This claim may be used only once. If a building contractor wishes to receive such compensation regularly, he must ply his trade regularly."

price r. If he does, he bids an amount r unopposed (or with token opposition—"rival" bids less than r). If he does not want it, the next bidder on the list is asked. This was how the electrical-equipment conspiracy operated, for example. The total expected profits from this mechanism are the same as from the identical-bidding mechanism, but in practice it requires more coordination, which creates a risk of cartel communications being detected but reduces the evidence given directly from bidding. Comanor and Schankman (1976) found that rotating-bid arrangements were significantly more common in cartels with few members than in cartels with many members, reflecting the extra difficulties of setting up and running a rotating-bid mechanism.

There are, however, two situations in which a cartel would prefer a rotating-bid mechanism to identical bidding. First, identical bidding results in each bidder winning with equal probability. Sometimes cartels seek a different division of the spoils from this. For example, in the electrical-equipment conspiracy, circuit-breaker contracts were allocated so that General Electric got 45 percent, Westinghouse got 35 percent, and Allis-Chalmers and Federal Pacific got 10 percent each (Smith, 1961 p. 137). Since total profits are linear in the shares, such a mechanism still yields maximal total expected profits.

The second reason for using rotating bids comes from the fact that, since the mechanism of Theorem 1 works by using the seller as a randomizing device, the seller can easily disrupt the mechanism by refusing to randomize. Instead of awarding the item arbitrarily when the bids are tied, the seller could announce a deterministic tie-breaking rule: for example, he could award the item to the smallest firm bidding, or to the bidder whose name comes first in the alphabet. Then it would no longer be in the interest of bidders so discriminated against to remain in the cartel. The cartel members could defeat this ploy by rotating their bids. Mund (1960) cites some instances of bidders for government contracts switching from an identical-bid mechanism to a rotating-bid mechanism after the government authorities became suspicious about the identical bids. There are indications that the U.S. government has recently begun using identical bidding as a basis for antitrust investigation.

IV. Strong Cartels

The cartel modeled in the last section was a relatively weak cartel. We now consider a stronger cartel, defined by two characteristics. First, it is able to exclude nonserious bidders (i.e., bidders who could never win in a noncooperative auction but who might be attracted by the cartel's profits), so it does not have to impose the no-entry constraint (6). Second, it is able to make transfer payments among its members. We shall find that side-payments enable the cartel to achieve efficiency, unlike in the case of no transfers (Theorem 1), where efficiency was achieved only in the trivial case in which collusion did no better than noncooperative behavior.[10]

An optimal cartel mechanism has the property that the bidder with the highest value wins if and only if his value exceeds r and the seller receives r. The optimal direct mechanism that implements this is as follows.

THEOREM 3: *The following mechanism is incentive-compatible and efficient. Before the auction, the cartel members report their valuations to the mechanism. If no report exceeds r, the cartel does not bid in the auction. If at least one bid exceeds r, the bidder making the highest report v obtains the item and pays a total of*

$$(7)\quad T(v) = F(v)^{-n} \times \int_r^v (u-r)(n-1)\,F(u)^{n-1} f(u)\,du + r.$$

Each losing bidder receives from the winner $[T(v)-r]/(n-1)$, and the seller receives r.

[10] That efficiency can be achieved is perhaps surprising in the light of the result of Myerson and Mark Satterthwaite (1983) that transfers do not guarantee efficiency in general bargaining situations when individual rationality is required.

This mechanism has the property that all losing bidders receive a transfer; in particular, and in contrast to the previous section's mechanisms, bidders whose value is less than the minimum price earn positive profit.

The cartel's optimal mechanism is not unique; there are many mechanisms yielding the same expected profit as a function of valuation. One mechanism corresponding to Theorem 3's mechanism has the cartel setting up an auction of its own.

COROLLARY: *The cartel can implement the mechanism of Theorem 3 by holding a prior first-price sealed-bid auction. If the highest bid in this prior auction exceeds r, the winner then bids r in the legitimate auction and pays each of the losers an equal share of the difference between his bid in the prior auction and r.*

To prove this corollary, note that, in the new mechanism, bidding $T(v)$ is an equilibrium, because, if all others bid $T(v)$, bidding $T(v)$ in the new mechanism coincides with responding honestly in the direct mechanism; also, $T(r) = r$, so the minimum price remains at r.

The mechanism described in the corollary is remarkably similar to a collusive technique commonly used in practice. One cartel member is arbitrarily assigned to bid for the item without competition from his fellow cartel members. Afterwards, the item is reauctioned among the cartel members (in the "knockout"). The cartel members then share a sum of money equal to the difference between the price reached in the cartel's own auction and the price reached in the legitimate auction. Such a practice has been described in auctions of antiques, books, fish, timber, industrial machinery, and wool by Ralph Cassady (1967 Ch. 13), Cooper (1977 pp. 35–8), Graham and Marshall (1987), and Arthur Halpern (1985).[11]

The only difference between this practice and the theoretical optimum is that the actual practice, by having the cartel's own auction after rather than before the legitimate auction, runs the risk of an inefficient outcome: the cartel might bid on an item and then discover that no member values it at more than the minimum price. It may, however, be possible to mitigate this inefficiency in practice: according to Halpern (1985 p. 3), a cartel often gives its designated bidder "authority to use his discretion in the bidding based upon his knowledge of the market for the desired item."[12]

THEOREM 4: *An efficient cartel mechanism has the property that the winner transfers to each of the losers an amount equal to*

$$(8) \qquad \pi(0) = \frac{E\left[v_{(2)} - r \mid v_{(1)} \geq r\right]}{n}$$

*where $v_{(j)}$ represents the j*th *order statistic and the expectation is taken over the distribution of the highest valuation. The winner's expected rent is this amount plus the rent he would have earned if the auction had been noncooperative.*

If the bidding were noncooperative, the total expected profit earned by the winning bidder would be the expected difference

[11]The first cartel convicted under the Sherman Antitrust Act of 1890, six cast-iron-pipe manufacturers, operated a knockout. In those cities not reserved for a particular firm, the price was fixed by a central committee (this is our r). Before a contract was let, the central committee accepted bids from the cartel members for the right to the contract. The lowest bidder then bid the prearranged price in the legitimate auction, and the others submitted higher bids. The surplus was periodically distributed to the cartel members in proportion to their production capacities (Stigler, 1966 p. 230).

[12]In India, according to Robert Wade (1987), the central government auctions the right to sell liquor in a village. Some villages (those that have succeeded in organizing for themselves a village committee) hold a prior auction among potential bidders. The winner then bids unopposed in the government auction. The village committee uses the difference between the village auction's price and the government auction's price to finance village-level public goods like the maintenance of wells. Evidently, the villagers have found the optimal mechanism.

between the highest valuation and the second-highest valuation (given the highest valuation), because the bidding would drive the price up to the second-highest valuation (see McAfee and McMillan [1987] for details). With collusion, the total expected profit earned by the cartel is the difference between the expected highest valuation and the minimum price (given that this difference is positive), because the price paid to the seller is the minimum price. Thus, the extra expected profit resulting from the cartel's activity, to be shared among the cartel members, is the difference between these two profit levels, or the expected second-highest valuation minus the minimum price. Theorem 4 has each of the cartel members getting an equal share of this sum (including the winner, whose total payment including his transfers to fellow cartel members is this amount less than what it would have been under noncooperative bidding).[13]

In practice, some cartels use sealed-bid auctions in the knockout (Halpern, 1985), but others use oral auctions (Cassady, 1967). However, in Theorem 4 it is essential that a first-price sealed-bid auction be used by the cartel; an oral auction would not work. This is because incentive compatibility requires that the payment to the losing bidders does not depend on their actual values. The price reached in an oral auction is the actual second value, whereas in a first-price auction it is the expected second value given the highest value, which depends only on the winner's value. In an oral knockout auction, as Cassady (1967 p. 182) and Stigler (1966 p. 231) noted, a bidder might overbid to raise the final price and thereby raise the transfer he receives if he loses: in other words, the oral knockout is not incentive-compatible. In particular, the use of the oral auction leads a bidder with value r to bid more than r, destroying efficiency. This would result in the winner paying more for the item that if the bidding had been noncooperative. In contrast, if the knockout uses a sealed-bid auction, losing bidders cannot raise the price by overbidding; instead they rationally bid the expected second-highest valuation conditional on their own valuation being the highest (as in an ordinary sealed-bid auction: see McAfee and McMillan [1987] for details), and thus the optimal mechanism of Theorem 3 is implemented.

In some situations, incentive constraints can be relaxed by eliminating the requirement that side-payments always sum to zero and just requiring that they sum to zero on average (cf. Theodore Groves, 1973; Bengt Holmstrom, 1982). In the case of a bidding cartel, this would allow the cartel to distribute the surplus gained from colluding as a lump sum, instead of as a random payment depending on values, thereby eliminating some of the incentive effects in bidding. However, Theorem 3 shows that efficiency can be achieved without breaking the budget; there is nothing to be gained by employing a third party as a budget-breaker. Nevertheless, there is one advantage to having a budget-breaker: the optimal mechanism can be implemented in dominant strategies. This is done by running the cartel's auction as a Vickrey auction and having the budget-breaker return the expected surplus to all bidders as a lump sum. It appears to be impossible for the cartel to implement an efficient outcome as a dominant-strategy equilibrium without breaking the budget (cf. Claude d'Aspremont and Louis-Andre Gérard-Varet, 1979).

V. Random Reserve Prices and Incomplete Cartels

What changes in our analysis if, instead of all bidders being in the cartel, only some of them are? A cartel that does not involve all the potential bidders may lose the bidding even when it bids in excess of the reserve price. The beginning point of the analysis of a partial cartel is formally equivalent to the analysis of a complete cartel facing a random reserve price. Suppose that the distribution of reserve prices, realized

[13]Graham and Marshall (1987) model the use of a knockout with all losing bidders receiving equal side-payments in English and Vickrey auctions. Theorem 4 supplements their analysis by showing that the knockout with equal profits is the optimal mechanism for the cartel: there is no other mechanism, no matter how complicated, that does better.

after the bidding takes place, is G; that is, if b is the highest bid submitted by the cartel, then b wins with probability $G(b)$. We consider the case of strong cartels. If v is the maximum value of the cartel, then the most the cartel can earn is

$$\max_b (v-b)G(b). \tag{9}$$

We let B_C maximize (9). A strong cartel can realize the profits in (9) using a prior auction, similar to the result in Theorem 3. The cartel holds a prior sealed-bid auction for the right to represent the cartel in the auction. The highest bidder in the prior auction pays his bid to the cartel, and this is divided evenly among the losing bidders. Facing no competition from other cartel members, the winning bidder will then choose to bid $B_C(v)$ in the legitimate auction. As is shown in the Appendix, if the cartel contains k members and there are $n-k$ nonmembers, the bid in the prior auction will be

$$\beta(v) = F(v)^{-k} \times \int_0^v [s - B_C(s)]G(B_C(s))(k-1)F(s)^{k-1}f(s)\,ds. \tag{10}$$

If the cartel is incomplete, nonmembers who know a cartel exists will, of course, react to the existence of the cartel. Suppose nonmembers use the increasing bidding function $B_N(v)$. Then, the probability that any one nonmember bids less than b is $F(B_N^{-1}(b))$. Thus,

$$G(b) = \begin{cases} 0 & b < r \\ F\left(B_N^{-1}(b)\right)^{n-k} & b \geq r. \end{cases}$$

Thus, a pure-strategy equilibrium is defined by the pair of equations

$$B_C(v) = \operatorname*{argmax}_{b \geq r} (v-b)F\left(B_N^{-1}(b)\right)^{n-k} \quad \text{for } v \geq r \tag{11}$$

$$B_N(v) = \operatorname*{argmax}_{b \geq r} (v-b)F\left(B_N^{-1}(b)\right)^{n-k-1} \times F\left(B_C^{-1}(b)\right)^{k} \quad \text{for } v \geq r. \tag{12}$$

Formally, (11) and (12) define an equilibrium when one bidder has a value drawn from the distribution F^k, and the other $n-k$ bidders have draws from the distribution F. It is not generally known when bidding equilibria exist in this environment. Because of the complexities of this environment, we now turn to a simpler model. Each potential bidder has a value $v \in \{0,1\}$, and $p = \Pr\{v=1\}$. As before, values are independently distributed. The reserve price r is in $(0,1)$. We use the notation Π_C^k, and Π_N^k, for the *ex ante* (before values are realized) profits of a cartel member and nonmember, respectively, when there are k members of a cartel. Bidders in this environment will randomize, and we consider equilibria in which all nonmembers use the same mixed strategy G_N. Should any cartel member have a high value, the cartel selects a representative to participate in the auction without competition from other cartel members. The following theorem provides a characterization of the equilibrium in this example.

THEOREM 5: *Suppose there are n bidders and k cartel members. Ex ante profits of nonmembers are*

$$\Pi_N^k = (1-r)(1-p)^{n-k}p$$

and exceed profits of members,

$$\Pi_C^k = (1-r)(1-p)^{n-k}(1/k)[1-(1-p)^k].$$

Nonmembers with high value choose a bid from the distribution

$$G_N(b) = \left[\left(\frac{1-r}{1-b}\right)^{1/(n-k)} - 1\right]\frac{1-p}{p} \quad \text{for } b \in (r, \bar{b}]$$

and the cartel representative bids according to the distribution

$$G_C(b) = \frac{(1-p)\left(\frac{1-r}{1-b}\right)^{1/(n-k)} - (1-p)^k}{1-(1-p)^k} \quad \text{for } b \in [r, \bar{b}]$$

if $k < n$, and submits a bid of r if $k = n$. Finally,

$$\bar{b} = 1 - (1 - r)(1 - p)^{n-k}.$$

It is interesting to note that $G_C(r) > 0$; that is, the cartel submits a bid r with positive probability. Moreover, the total profits of the cartel, conditional on a high value, equal the expected profits of any nonmember who has a high value. However, a larger cartel produces greater per capita profits:

$$(13) \qquad (k+1)\Pi_C^{k+1} > k\Pi_C^k + \Pi_N^k.$$

Consider the following cartel-formation game. All bidders are simultaneously asked if they would like to join the cartel. Those who respond "yes" are members, while those saying "no" are nonmembers, and the auction bidding game is played. A pure-strategy equilibrium to the simultaneous cartel formation game is defined by

$$(14) \qquad \Pi_C^k \geq \Pi_N^{k-1}$$

$$(15) \qquad \Pi_C^{k+1} \leq \Pi_N^k.$$

That individuals slated by the equilibrium to be cartel members wish to join is guaranteed by (14), and similarly that nonmembers do not wish to join is guaranteed by (15). Conditions (14) and (15) define an equilibrium cartel size.

THEOREM 6: *An equilibrium to the cartel-formation game has k^* cartel members if*

$$\frac{1-(1-p)^{k^*}}{k^*} \geq p(1-p) \geq \frac{1-(1-p)^{k^*+1}}{k^*+1}.$$

For almost every p, k^* is unique. $k^* \geq 3$, k^* is nondecreasing for $p > 0$, and $k^* \to \infty$ as $p \to 1$.

Cartels, in the simultaneous-formation game, have at least three members, and the number of members is independent of the number of potential members, although of course $k^* \leq n$.

A somewhat different formation game leads to a cartel of the whole set of bidders. Suppose the cartel sequentially goes through the set of potential bidders, offering a side-payment s_i to bidder i to join the cartel, and if a bidder refuses to join the cartel, no one else is asked to join. This procedure leads to the cartel of the whole, by (13), which guarantees that there is always a sufficient side-payment. Of course, the rule that the cartel stops growing as soon as any bidder refuses to join is difficult to justify.

A natural extension of this analysis would permit multiple cartels. However, this creates difficulties in constructing the bidding equilibrium, because it appears that smaller cartels will no longer randomize their bids with the same support as the larger cartels.

We have been unable to characterize the optimal strategy for a weak cartel. However, consider the weak cartel facing a random reserve price with distribution G. Provided G is concave, the coordinated cartel selects a member to be a representative, and then this member bids $B_C(v_i)$ if he has value v_i. While this result is useful for the random reserve price, it appears less promising for the case of an incomplete cartel, for G is endogenous. In addition, the problems of existence of bidding functions for the strong cartel seem likely to arise with weak cartels as well.[14]

VI. The Seller's Response

Whether the cartel is weak or strong, the effect on the seller is the same: the highest bid submitted equals the minimum price.[15]

[14]On collusion by subsets of bidders in English and second-price auctions, see Graham and Marshall (1987) and Malaith and Zemsky (1991).

[15]Collusion is common in the U.S. Forest Service's auctions of federally owned timber, because transportation costs mean that often only local mills bid (W. J. Mead, 1967). For instance, in the ponderosa pine region in the early 1980's, in 30 percent of the auctions the winning bid was within 1 percent of the reserve price. In some of these auctions only local firms bid; in others, there was competition from outside firms. Consistent with the hypothesis that outside competition destroys collusion among the local firms, in those auctions in which outsiders bid, the winning bid was 2–3 times the reserve price (Mead et al., 1983).

Suppose the seller can somehow recognize that he faces organized bidders. What strategies can the seller use to counter the cartel? We shall discuss three types of strategy: raising the reserve price, keeping the reserve price secret, and interfering with the cartel's enforcement mechanism.

What is the seller's objective function? If the seller is the government, concerned only with efficiency, then, to the extent that collusive profits represent a pure transfer, the seller is indifferent about the cartel's existence. Indeed, in this case, the government should prefer a strong cartel (using transfer payments) to a weak cartel, for this ensures the efficiency of the outcome (cf. Theorems 1 and 3). However, if government revenue is raised by distortionary taxation, as is realistically the case, then the collusive profits earned by the cartel cause a welfare loss: the government/seller would be concerned about the cartel's profits. Similarly, if the seller is a risk-neutral individual or firm, the objective would be to maximize own expected surplus. Let us assume from here on that this is the case.

The seller can set the reserve price so as to mitigate the effects of the cartel (assuming he knows the number of cartel members). Let $J(v) = v - [1 - F(v)]/f(v)$ and assume the standard regularity condition: $J(v)$ is strictly increasing.[16] The seller's expected return is $(r - v_0)[1 - F(r)^n]$, where v_0 is the seller's return if he fails to sell the item. Maximization of this over r yields the optimal anticartel reserve price r_c, satisfying

$$(16)\quad r_c - v_0 - \frac{1 - F(r_c)^n}{nF(r_c)^{n-1} f(r_c)} = 0.$$

This is a special case of a result of Graham and Marshall (1987). The reserve price implied by (16) is higher than the optimal reserve price when the bidding is noncooperative, r_n, defined by

$$(17)\quad r_n - v_0 - \frac{1 - F(r_n)}{f(r_n)} = 0$$

(Myerson, 1981; John Riley and William Samuelson, 1981). Thus, upon discovering that the bidders are colluding, the seller raises his reserve price, setting it higher the larger is the number of cartel members.

The imposition of the anticartel reserve price r_c lowers the profits of the bidders. Indeed, if there are few bidders, collusive profits with the reserve price r_c may be lower than noncooperative profits with the reserve price r_n. This is illustrated in Table 1, for F uniform on $[0, 1]$ and v_0 either 0 or 0.5. Profits under noncooperative bidding, implicit collusion, and explicit collusion are listed for various numbers of bidders, n. Note that, for $v_0 = 0$, there must be at least nine bidders before detectable implicit collusion becomes profitable; for $v_0 = 0.5$, there must be 19 or more bidders.

This example suggests that the seller's choice of action will affect bidding behavior by both collusive and noncollusive bidders. If the discount rate is low enough, an industry with few firms will have an incentive not to collude, because current gains will be outweighed by lost future profits after the seller detects the collusion and raises the reserve price to its anticartel level. Paradoxically, Table 1 suggests that, if the seller adjusts the reserve price optimally, collusion is more likely in industries with many firms than in industries with few firms. Also, the method by which the seller deduces that he faces a cartel (e.g., by observing a sequence of bids near the reserve price) will affect the decision of the bidders on whether or not to collude. The threat of a higher reserve price may be enough to deter collusion. However, the dynamic equilibrium involving the threat of a raised reserve price and the bidders' optimizing responses to it is beyond the scope of this paper.

Our analysis assumed that all bidders know the minimum price r. In practice, reserve prices are often kept secret; this is sometimes explained as an anticartel device.

[16] This condition ensures that the seller does not use a stochastic policy (see Myerson, 1981; Eric Maskin and Riley, 1984; McAfee and McMillan, 1987). In terms of the cases considered in Theorem 2, the assumption on J implies that $H'(v) < 1$; thus, the assumption underlying Theorem 2's identical-bidding result is weaker than the standard regularity condition.

TABLE 1—COLLUSIVE PROFITS AND RESERVE PRICES

	$v_0 = 0$, $r_n = 0.5$				$v_0 = 0.5$, $r_n = 0.75$			
n	r_c	π_i	π_e	π_n	r_c	π_i	π_e	π_n
2	0.58	0.070	0.077	0.083	0.77	0.024	0.025	0.026
3	0.63	0.046	0.053	0.057	0.78	0.019	0.020	0.022
4	0.67	0.033	0.039	0.041	0.80	0.015	0.017	0.018
5	0.70	0.025	0.031	0.030	0.81	0.013	0.014	0.016
6	0.72	0.020	0.025	0.022	0.82	0.011	0.012	0.013
7	0.74	0.016	0.020	0.017	0.83	0.009	0.011	0.011
8	0.76	0.013	0.017	0.014	0.84	0.007	0.010	0.010
9	0.77	0.011	0.015	0.011	0.84	0.007	0.008	0.008
10	0.79	0.010	0.013	0.009	0.85	0.006	0.007	0.007
15	0.83	0.005	0.007	0.004	0.88	0.004	0.005	0.004
20	0.86	0.003	0.005	0.002	0.89	0.002	0.003	0.002

Notes: F is uniform on $[0,1]$. n = number of bidders; r_c = anticartel reserve price; π_i = implicitly collusive profit per firm (with reserve price r_c); π_e = explicitly collusive profit per firm (with reserve price r_c); and π_n = noncooperative profit per firm (with reserve price r_n).

Should the seller announce r? There are two cases. First, suppose the seller's valuation v_0 is common knowledge and it is common knowledge whether the seller perceives that a cartel exists, so that all bidders know which of (16) and (17) apply. Then the bidders can deduce r even if it is not announced: the seller is indifferent between announcing and not announcing. Second, suppose v_0 is not common knowledge, or it is not common knowledge which of (16) or (17) the seller will use. Then there is one sense in which the seller gains by keeping r secret. For the bidders to be able to coordinate their bids, they must agree in their estimates of r. Not announcing r forces the bidders to communicate in order to determine the level of the winning bid. The need for communication may make the cartel more fragile (cf. Section III).

Suppose the cartel uses trigger strategies as its enforcement mechanism, reverting to noncooperative behavior in some or all future auctions if a nondesignated bidder wins. Then, as a general rule, the seller should make the environment as stochastic as possible for the cartel, so as to cause the trigger to be set off accidentally: for example, the seller might occasionally choose the wrong bidder (provided he does not reveal the winner's bid). Also, as Milgrom (1987) argued, trigger strategies work better in oral auctions than in sealed-bid auctions, for the punishment for deviation comes immediately, instead of in the next auction. Thus, a seller concerned about possible collusion should choose a sealed-bid auction rather than an oral auction. Finally, the larger the prize, the greater the temptation to defect from the cartel, because the effective discount rate is larger. In the case of contract bidding, this implies that it is better to offer a project as a single large contract than to break it up into several smaller contracts.

VII. Conclusion

We have modeled bidding cartels with and without side-payments among members. In each case, the effect on the seller is the same: just as if there were only one bidder, the price paid is the minimum price. From the bidders' point of view, however, there are differences. With transfers, efficiency is achieved so that the bidders' expected profit is maximized. Without transfers, efficiency is not attainable; the winner is arbitrarily selected from those bidders who value the item at more than the minimum price.

APPENDIX

PROOF OF THE LEMMA:

In the noncooperative equilibrium of a first-price sealed-bid auction with reserve price r, it is well known that an agent with value v bids

$$(A1)\quad B(v) = v - F(v)^{-(n-1)}\int_r^v F(u)^{n-1}\,du$$

(Myerson, 1981; Riley and Samuelson, 1981; Milgrom and Weber, 1982; McAfee and McMillan, 1987).

Since the probability of winning is $F(v)^{n-1}$, this yields expected profit of $\int_r^v F(u)^{n-1}\,du$, so that, by integration by parts, *ex ante* expected profits (before values are known) are

$$(A2)\quad E\pi = \int_r^{v_h}[1-F(u)]F(u)^{n-1}\,du.$$

Now, incentive compatibility (2), efficiency (4), and zero profits for values at or below the reserve price imply that expected profits are

$$(A3)\quad \int_0^{v_h}\pi_i(u)f(u)\,du = \int_r^{v_h}\pi_i(u)f(u)\,du$$

$$= -\pi_i(u)[1-F(u)]\Big|_r^{v_h} + \int_r^{v_h}\frac{d\pi_i}{du}[1-F(u)]\,du$$

$$= \int_r^{v_h}[1-F(u)]F(u)^{n-1}\,du$$

the same as noncooperative profit (A2).

PROOF OF THEOREM 1:

Because transfers are impossible and the minimum acceptable bid is r, any bidder with value less than r must make nonpositive profit. It follows that any scheme that allows bidders with values less than r to win is dominated by one in which they lose, as

$$(A4)\quad \int_0^{v_h}\pi_i(v)f(v)\,dv \le \int_0^r 0f(v)\,dv + \int_r^{v_h}\pi_i(v)f(v)\,dv.$$

Thus, expected profits are

$$(A5)\quad E\pi_i(v) = \int_r^{v_h}\pi_i(v)f(v)\,dv = -\pi_i(v)[1-F(v)]\Big|_r^{v_h} + \int_r^{v_h}[1-F(v)]E_{-i}h_i(v_i,\mathbf{v}_{-i})\,dv.$$

Thus, the sum of expected profits is

$$(A6)\quad E\sum_{i=1}^n \pi_i(v) = E\sum_{i=1}^n H(v_i)h_i(v_i,\mathbf{v}_{-i}).$$

In the case $H'(v)\ge 0$, the profit-maximizing scheme has the highest-value bidder winning if and only if his value exceeds r. By the lemma, this results in noncooperative profits. In the case $H'(v)<0$, the optimal collusive scheme maximizes total expected profits (A6) subject to

$$(A7)\quad v_i < r \rightarrow h_i(v_i,\mathbf{v}_{-i}) = 0$$

$$(A8)\quad 0 \le h_i(v_i,\mathbf{v}_{-i})$$

$$(A9)\quad \sum_{i=1}^n h_i(v_i,\mathbf{v}_{-i}) \le 1$$

$$(A10)\quad \frac{\partial}{\partial v_i}E_{-i}h_i(v_i,\mathbf{v}_{-i}) \ge 0.$$

From (A7) and (A9), we have

$$(A11)\quad E\sum_{i=1}^n h_i(v_i,\mathbf{v}_{-i}) \le 1-[F(r)]^n.$$

Let $\mu_i(v_i) = E_{-i}h_i(v_i, \mathbf{v}_{-i})$. Then, total expected profits are

$$\text{(A12)}\quad \sum_{i=1}^{n}\int_r^{v_h}\left[\frac{1-F(v_i)}{f(v_i)}\right]\mu_i(v_i)f(v_i)\,dv_i$$

$$= \sum_{i=1}^{n}\Bigg([1-F(r)] \times \int_r^{v_h}\left[\frac{1-F(v_i)}{f(v_i)}\right]\mu_i(v_i)\left[\frac{f(v_i)}{1-F(r)}\right]dv_i\Bigg)$$

$$\le \sum_{i=1}^{n}\Bigg([1-F(r)] \times\left[\int_r^{v_h}\left(\frac{1-F(v_i)}{f(v_i)}\right)\left(\frac{f(v_i)}{1-F(r)}\right)dv_i\right] \times\left[\int_r^{v_h}\mu_i(v_i)\left(\frac{f(v_i)}{1-F(r)}\right)dv_i\right]\Bigg)$$

$$= \left[\int_r^{v_h}\left(\frac{1-F(v)}{1-F(r)}\right)dv\right] \times\left[\int_r^{v_h}\sum_{i=1}^{n}\mu_i(v_i)f(v_i)\,dv_i\right]$$

$$= \left[\int_r^{v_h}\left(\frac{1-F(v)}{1-F(r)}\right)dv\right]E\sum_{i=1}^{n}h_i(v_i,\mathbf{v}_{-i})$$

$$\le \left[\int_r^{v_h}\left(\frac{1-F(v)}{1-F(r)}\right)dv\right](1-[F(r)]^n).$$

The first inequality holds since $[1-F(v_i)]/f(v_i)$ is decreasing and $\mu_i(v_i)$ is nondecreasing, (and thus the expected value of the product does not exceed the product of the expected values). The second inequality follows from (A11).

One implementation of the bound on profits in (A12) is for all bidders with values in excess of r to bid r, since profits are:

$$\pi_i = \left[\int_r^{v_h}(v-r)f(v)\,dv\right] \times\sum_{j=0}^{n-1}\left[\frac{1}{j+1}\binom{n-1}{j} \times[1-F(r)]^j[F(r)]^{n-j-1}\right]$$

$$= \left[\int_r^{v_h}[1-F(v)]\,dv\right]\frac{1}{n} \times\sum_{j=0}^{n-1}\left[\binom{n}{j+1} \times[1-F(r)]^j[F(r)]^{n-j-1}\right]$$

$$= \left[\int_r^{v_h}[1-F(v)]\,dv\right]\frac{1}{n[1-F(r)]} \times\sum_{k=1}^{n}\binom{n}{k}[1-F(r)]^k[F(r)]^{n-k}$$

$$= \frac{1}{n}\int_r^{v_h}\left[\frac{1-F(v)}{1-F(r)}\right]dv\left(1-[F(r)]^n\right)$$

and summing achieves the bound.

PROOF OF THEOREM 2:

It is sufficient to show that maximum expected profits satisfy (A6). From (2) and (6),

$$\text{(A13)}\quad \int_0^{v_h}\pi_i(v)f(v)\,dv$$

$$= \int_r^{v_h}\pi_i(v)f(v)\,dv$$

$$= -\pi_i(v)[1-F(v)]\Big|_r^{v_h} + \int_r^{v_h}[1-F(v)]E_{-i}h_i(v_i,\mathbf{v}_{-i})\,dv$$

$$= E\sum_{i=1}^{n}\frac{1-F(v_i)}{f(v_i)}h_i(v_i,\mathbf{v}_{-i}).$$

That is, the constraint (6) means that the cartel maximizes the same objective function subject to the same constraints as in Theorem 1.

PROOF OF THEOREM 3:

Suppose that the mechanism described in the theorem is operating and suppose that a bidder with value v reports w. Since a report of $w < r$ will never win and hence results in constant profit, we need only consider the case $w \geq r$. From (7), the bidder gets expected profits of

$$\text{(A14)}\quad \begin{aligned}\pi &= [v - T(w)]F(w)^{n-1} \\ &\quad + [1 - F(w)^{n-1}] \\ &\quad \times \int_w^{v_h} \left[\frac{T(u) - r}{n-1}\right] \frac{(n-1)F(u)^{n-2}f(u)}{1 - F(w)^{n-1}}\, du \\ &= [v - T(w)]F(w)^{n-1} \\ &\quad + \int_w^{v_h} [T(u) - r]F(u)^{n-2}f(u)\, du.\end{aligned}$$

Thus,

$$\text{(A15)}\quad \begin{aligned}\frac{\partial \pi}{\partial w} &= [v - T(w)](n-1)F(w)^{n-2}f(w) \\ &\quad - T'(w)F(w)^{n-1} \\ &\quad - [T(w) - r]F(w)^{n-2}f(w) \\ &= [(n-1)v - nT(w) + r]F(w)^{n-2}f(w) \\ &\quad - T'(w)F(w)^{n-1}.\end{aligned}$$

Since $\partial^2 \pi / \partial v\, \partial w \geq 0$, incentive compatibility is characterized by

$$\text{(A16)}\qquad \left.\frac{\partial \pi}{\partial w}\right|_{w=v} = 0.$$

Now, from (7),

$$\text{(A17)}\quad \left.\frac{\partial \pi}{\partial w}\right|_{w=v}$$

$$\begin{aligned}&= \Big[(n-1)v + r - nF(v)^{-n} \\ &\quad \times \int_r^v (u - r)(n-1)F(u)^{n-1}f(u)\, du - nr\Big] \\ &\quad \times F(v)^{n-2}f(v) \\ &\quad - F(v)^{n-1}\Big[-nF(v)^{-(n+1)}f(v) \\ &\quad\quad \times \int_r^v (u - r)(n-1)F(u)^{n-1}f(u)\, du \\ &\quad\quad + F(v)^{-n}[(v - r)(n-1)F(v)^{n-1}f(v)]\Big] \\ &= (n-1)vF(v)^{n-2}f(v) - nF(v)^{-2}f(v) \\ &\quad \times \int_r^v (u - r)(n-1)F(u)^{n-1}f(u)\, du \\ &\quad - (n-1)rF(v)^{n-2}f(v) + nF(v)^{-2}f(v) \\ &\quad \times \int_r^v (u - r)(n-1)F(u)^{n-1}f(u)\, du \\ &\quad - (n-1)(v - r)F(v)^{n-2}f(v) \\ &= 0.\end{aligned}$$

Thus, the mechanism is incentive-compatible; it is clearly efficient.

PROOF OF THEOREM 4:

Note that, from incentive compatibility,

$$\text{(A18)}\qquad \frac{d\pi}{dv} = F(v)^{n-1} \qquad \text{if } v \geq r.$$

Thus,

$$\text{(A19)}\quad \pi(v) = \begin{cases} \pi(0) + \int_r^v F(u)^{n-1}\, du & \text{if } v \geq r \\ \pi(0) & \text{if } v < r \end{cases}$$

where $\pi(0)$ is the transfer received by each losing bidder.

Moreover, the total rent earned by the cartel is the expected difference between

the winner's value and r, where the density function of the winner's value is $nF(v)^{n-1}f(v)$. This total rent must equal total expected profits as in (A20):

$$\text{(A20)}\quad \frac{1}{n}\int_r^{v_h}(v-r)nF(v)^{n-1}f(v)\,dv$$

$$=\int_0^{v_h}\pi(v)f(v)\,dv$$

$$=\pi(0)+\int_r^{v_h}\int_r^{v}F(u)^{n-1}\,du f(v)\,dv$$

$$=\pi(0)+\int_r^{v_h}[1-F(v)]F(v)^{n-1}\,dv.$$

Hence, the transfer to losing bidders is

$$\text{(A21)}\quad \pi(0)$$

$$=\int_r^{v_h}(v-r)F(v)^{n-1}f(v)\,dv$$

$$-\int_r^{v_h}[1-F(v)]F(v)^{n-1}\,dv$$

$$=\frac{1}{n}\int_r^{v_h}\left[v-\frac{1-F(v)}{f(v)}-r\right]nF(v)^{n-1}f(v)\,dv$$

$$=\frac{1}{n}E[v_{(2)}-r\,|\,v_{(1)}\geq r]$$

where $v_{(j)}$ represents the jth order statistic. In a first-price sealed-bid auction, bids equal the expected second value conditional on own value. Since r is the price paid in the legitimate auction, the transfer to losers $\pi(0)$ can, from (A21), be interpreted as $(1/n)$th of the difference between the winning bid in the cartel's sealed-bid auction and the bid in the legitimate auction.

Derivation of (10): Consider a bidder with value v who bids $\beta(\hat{v})$ in the prior auction. He obtains

$$\pi=\{[v-B_C^*(v)]G(B_C^*(v))-\beta(\hat{v})\}F(\hat{v})^{k-1}$$

$$+\int_{\hat{v}}^{v_h}\frac{\beta(s)}{k-1}(k-1)F(s)^{k-2}f(s)\,ds$$

$$\frac{\partial\pi}{\partial\hat{v}}=\left[(v-B_C^*(v))G(B_C^*(v))-\frac{k}{k-1}\beta(\hat{v})\right]$$

$$\times(k-1)F(\hat{v})^{k-2}f(\hat{v})-F(\hat{v})^{k-1}\beta'(\hat{v})$$

$$\frac{\partial^2\pi}{\partial v\,\partial\hat{v}}=G(B_C^*(v))(k-1)F(\hat{v})^{k-2}f(\hat{v})>0.$$

Thus, the solution to $0=\partial\pi/\partial v|_{\hat{v}=v}$ is an equilibrium. This linear differential equation has solution (10).

PROOF OF THEOREM 5:

Suppose the cartel, composed of k firms, bids with the distribution G_C, and $n-k$ noncartel members bid with distribution G_N. Cartel profits are, for bid $b\geq r$, provided the cartel has a 1,

$$\pi_C=(1-b)\left[1-p+pG_N(b)\right]^{n-k}$$

and nonmembers with a 1 obtain

$$\pi_N=(1-b)[1-p+pG_N(b)]^{n-k-1}$$

$$\times\{(1-p)^k+[1-(1-p)^k]G_C(b)\}.$$

Standard arguments insure that G_N and G_C have no mass points in the interior of their support or at the right endpoint of the support, nor do they have any gaps. Moreover, $\pi_C=\pi_N$. To see this, let $\bar{b}$ be the maximum nonmember bid. The maximum cartel bid cannot exceed $\bar{b}$, or else the cartel obtains higher profits by bidding $\bar{b}$. Therefore $G_N(\bar{b})=G_C(\bar{b})=1$, and $\pi_N=1-\bar{b}$, and $\pi_C\geq 1-\bar{b}$. Now suppose $G_C(\bar{b}_C)$

$= 1$, for $\bar{b}_C < \bar{b}$. Then, for $b \in (\bar{b}_C, \bar{b})$,

$$\pi_C = (1-b)[1-p+pG_N(b)]^{n-k}$$
$$= \pi_N[1-p+pG_N(b)]$$
$$< \pi_N$$

a contradiction. Thus, $\pi_N = \pi_C = 1-\bar{b}$.

If there are at least two noncartel members, $G_N(r) = 0$, for otherwise a slight increase in bid brings a discrete increase in the probability of winning. For the same reason, $G_N(r)G_C(r) = 0$. If $G_C(r) = 0$, let b_C be the left endpoint of G_C's support. Then,

$$\pi_C = (1-b_C)[1-p+pG_N(b_C)]^{n-k}$$
$$< (1-b_C)[1-p+pG_N(b_C)]^{n-k-1}$$
$$= \pi_N$$

a contradiction. Thus $b_C = r$. Even with $n-k=1$, $G_N(r) = 0$, since $G_N(r) > 0$ implies $G_C(r) = 0$, and

$$\pi_C = (1-r)(1-p)$$
$$> (1-r)(1-p)^{n-1}$$
$$= \pi_N$$

a contradiction.

$G_N(r) = 0$ shows

$$\pi_N = \pi_C = (1-r)(1-p)^{n-k}$$

which yields

$$G_N(b) = \left[\left(\frac{1-r}{1-b}\right)^{1/(n-k)} - 1\right]\frac{1-p}{p} \quad \text{for } r \le b \le \bar{b}$$

$$G_C(b) = \frac{\left(\frac{1-r}{1-b}\right)^{1/(n-k)}(1-p) - (1-p)^k}{1-(1-p)^k} \quad \text{for } r \le b \le \bar{b}$$

$$\bar{b} = 1-(1-r)(1-p)^{n-k}.$$

Hence,

$$G_C(r) = \frac{1-p-(1-p)^k}{1-(1-p)^k} > 0.$$

The *ex ante* profit of a nonmember is

$$\text{(A22)} \quad \Pi_N^k = \pi_N p = (1-r)(1-p)^{n-k}p$$

and the *ex ante* profit of a cartel member is

$$\text{(A23)} \quad \Pi_C^k = \pi_C \frac{1-(1-p)^k}{k}$$
$$= (1-r)(1-p)^{n-k}\frac{1-(1-p)^k}{k}.$$

$\Pi_N^k \ge \Pi_C^k$, because $(1-p)^k + kp \ge 1$ (the left-hand side is 1 at $p=0$, with derivative $k[1-(1-p)^{k-1}] \ge 0$).

PROOF OF THEOREM 7:

Substitution of (A22) and (A23) into (14) and (15) yield the first statement. Since $[1-(1-p)^k]/k$ is decreasing in k, there is a unique k^* associated with a stable cartel. Since $1-(1-p)^3 = 3p(1-p)+p^3$, $k^* \ge 3$.

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