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# Bidding for contracts: a principal-agent analysis

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*This article models the process of bidding for government contracts in the presence of moral hazard. Several (possibly risk-averse) potential contractors (agents) submit sealed bids, on the basis of which the government (principal) selects one to perform a task. The optimal linear contract is derived. The bidding process induces the potential agents to reveal their relative expected costs. The optimal contract trades off giving the chosen agent an incentive to limit costs against stimulating bidding competition and sharing risks. The optimal contract is never cost-plus, may be fixed-price, but is usually an incentive contract. Some prescriptions for government contracting emerge.*

## 1. Introduction

■ When the government offers a contract for a project such as the construction of a road or a warship, it usually calls for bids from interested firms and selects the lowest bidder. There are several informational asymmetries in this process. The government cannot directly observe any bidder's expected production costs, and therefore it does not know which is the efficient firm. Each bidder must determine his bid in ignorance of the expected costs of his rivals. After a bidder has been selected, he is better informed than the government about the vagaries of the particular project; thus, the government is unable to observe how much effort the firm is making to limit production costs.

The government must design a contract to address both adverse selection (the government does not know the expected cost of any firm) and moral hazard (the government cannot observe the selected firm's effort to keep its realized production costs low). To complicate matters, if the firms are risk averse, it is in the government's interest to offer a contract in which the government bears some of the risk of unpredictable cost fluctuations.

The forms of contract used in practice by governments make the payment to the contractor a linear function of its bid and/or its realized costs. With a fixed-price contract, the payment is simply the firm's bid. With a cost-plus contract, the government agrees to cover completely the costs incurred by the contractor, plus pay a fee that is either fixed in advance or is a proportion of costs. An incentive contract makes the payment depend both on the bid and on realized costs: if realized costs exceed the firm's bid, the firm is responsible for

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some fraction of the cost overrun; if the firm succeeds in holding its costs below its bid, it is rewarded by being allowed to keep part of the cost underrun.

This article solves for the linear contract that is optimal for the government. We show that the optimal linear contract is determined by a tradeoff between stimulating competition in the initial bidding for the contract and sharing risk between the contractor and the government, on the one hand, and giving the contractor incentives to limit his production costs, on the other.

The form of contract most commonly used by governments is the fixed-price contract, although cost-plus contracts are sometimes used, and incentive contracts are increasingly used by the United States Department of Defense for weapons acquisition.<sup>1</sup> The results of this article suggest that fixed-price contracts should be used much less frequently than they currently are and that cost-plus contracts should not be used if there is more than one bidder. Usually, the contract that minimizes procurement costs is an incentive contract. We shall show that it is feasible in practice to compute the parameters of the optimal linear incentive contract.

## 2. The linear contract

■ We assume that there are  $n$  ( $n > 1$ ) agents capable of performing a task. Given that agent  $i$  is selected, the *ex post* cost of the project,  $c_i$ , has three components:

$$c_i = c_i^* + w - \xi, \quad (1)$$

where  $c_i^*$  is the  $i$ th agent's expected (opportunity) cost of the project,  $w$  is a random variable representing unpredictable costs that arise in the course of the project, and  $\xi$  represents the extent to which actual costs are reduced by the agent's effort.

The value of  $c_i^*$ , which is known to agent  $i$  but not to the principal nor to the other agents, is an independent realization<sup>2</sup> of a distribution  $G(c_i^*)$ . Let  $g = G'$ , and let  $c_l$  be the lowest possible cost and  $c_h$ , possibly infinite, the highest possible cost (more precisely  $c_l \equiv \inf \{c | g(c) > 0\}$  and  $c_h \equiv \sup \{c | g(c) > 0\}$ ). Assume  $c_h > c_l$ .

All agents are assumed to face the same distribution of unpredictable costs  $F(w)$ ; let  $f = F'$ . We also assume, without loss of generality, that the expected value of  $w$  is zero.

Let  $h(\xi)$  represent the dollar cost to the agent of his effort to reduce actual costs. This amount  $h(\xi)$  cannot be charged to the project. The assumption that the agent bears the full burden of the cost-reduction effort is without loss of generality, for any cost-reduction innovation that can be charged to the project may be directly embedded in  $c_i^*$ . Assume  $h'' > 0$ , so that there are decreasing returns to cost-reducing activities. The agent's effort,  $\xi$ , cannot be observed by the principal.<sup>3</sup>

The principal, being risk neutral, is assumed to design a contract to minimize his expected payment to the agent. We also assume that the contract makes the payment,  $P$ , contingent on both the *ex post* cost of the project,  $c$ , and the successful agent's bid,  $b$ , and that, like the contracts used in practice, the contract is linear:<sup>4</sup>

$$P = \alpha c + \beta b + \gamma, \quad (2)$$

<sup>1</sup> On the use of the different contract forms in practice, see DeMayo (1983), Fox (1974), and McAfee and McMillan (1986b).

<sup>2</sup> This is an independent-private-values model in the terminology of Milgrom and Weber (1982).

<sup>3</sup> The nature of the cost-reduction possibilities is discussed in some detail by Scherer (1964).

<sup>4</sup> Note that in general the principal could do better by using a nonlinear contract. In Laffont and Tirole (1985) and McAfee and McMillan (1986a), it is shown for the special case of risk-neutral bidders that the optimal contract is linear in realized costs, but nonlinear in bid. With risk-averse bidders there is another sense in which the contract (2) is suboptimal: as Maskin and Riley (1984) and Matthews (1983) show, with risk aversion there are gains from requiring payments from some of the unsuccessful bidders and subsidizing others.

for some constants  $\alpha$ ,  $\beta$ , and  $\gamma$ . If  $\alpha = 1$  and  $\beta = 0$ , (2) defines a cost-plus contract and  $\gamma$  then represents the profit. If  $\alpha = 0$  and  $\beta = 1$ , this is a fixed-price contract. If  $0 < \alpha < 1$  and  $\beta = 1 - \alpha$ , this is an incentive contract, where the agent is responsible for a fraction  $(1 - \alpha)$  of any cost overrun.<sup>5</sup> Note that in the standard auction-theory formulation  $\alpha = \gamma = 0$  and  $\beta = 1$ .

Although the contract (2) appears to give the principal three parameters to control ( $\alpha$ ,  $\beta$ , and  $\gamma$ ), the only parameter of consequence is the share ratio  $\alpha$ . If  $\alpha < 1$ , then the contractor's realized costs are not entirely covered by the principal. As a result, the higher a firm's expected cost, the higher it will be forced to bid. Thus, bids reveal relative expected costs: by selecting the lowest bidder, the principal selects the most efficient firm. Conversely, under a cost-plus contract ( $\alpha = 1$ ) there is no reason for a high-cost firm to bid higher than a low-cost firm:<sup>6</sup> a cost-plus contract can never be optimal if there is more than one bidder because the principal will with probability  $(n - 1)/n$  fail to select the firm with the lowest expected costs. We therefore limit attention to the case  $\alpha < 1$ .

The parameters  $\gamma$  and  $\beta$  are inconsequential. Any values of  $\gamma$  will result in the same payment, because, if two or more bidders compete for the contract, an increase in  $\gamma$  would induce an equal decrease in each bid. For the same reason, any strictly positive value of  $\beta$  results in the same payment; the proviso that  $\beta$  be positive is needed for the payment to be positively related to the winning bid. Thus, to economize on notation without loss of generality, we set  $\gamma = 0$  and  $\beta = (1 - \alpha)$ . Now the contract (2) can be written as  $P = b + \alpha(c - b)$ ; that is, payment equals bid plus a fraction  $\alpha$  of the cost overrun or underrun.

In Section 3 we solve for each agent's expected-utility-maximizing choice of  $b$  and  $\xi$ , for any given linear contract. Then in Section 4 we find the principal's expected-payment-minimizing choice of contract, given the potential agents' optimizing responses. Section 5 investigates comparative statics for a special case.

### 3. Potential agents' optimization

■ We assume that the potential agents may be risk averse and that all potential agents have the same utility function  $U$ . If selected, agent  $i$  chooses a level of effort  $\xi_i$  to maximize expected utility of profit,  $EU(\pi_i)$ , where the expectation is taken over  $w$ . Profit  $\pi_i$  is

$$\begin{aligned}\pi_i &= \alpha c_i + (1 - \alpha)b_i - c_i - h(\xi_i) \\ &= (1 - \alpha)(b_i - c_i^* - w) + k_i,\end{aligned}\tag{3}$$

where

$$k_i = (1 - \alpha)\xi_i - h(\xi_i).\tag{4}$$

If  $EU(\pi_i) < 0$ , the agent does not bid.

Assume that an agent chooses  $\xi_i$  only after being awarded the contract; he may choose it either before or after observing  $w$ . Then, if agent  $i$  is awarded the contract,  $\xi_i$  will be chosen to satisfy

$$0 = EU'(\pi_i)(1 - \alpha - h'(\xi_i)),\tag{5}$$

from which

$$\xi_i = h'^{-1}(1 - \alpha).\tag{6}$$

<sup>5</sup> In an alternative type of cost-plus contract, profit is a percentage of realized costs so that  $\alpha > 1$ . The failings of the contract with  $\alpha = 1$ , to be discussed in what follows, are magnified in the contract with  $\alpha > 1$ . In United States military contracting, two types of incentive contracts are used: fixed-price-incentive and cost-plus-incentive-fee. The difference between the two is that the former includes a ceiling price that sets the maximum amount the government need pay (Fox, 1974, chap. 11). The analysis to follow will not distinguish between these two types of incentive contracts.

<sup>6</sup> How firms would bid in a cost-plus contract, that is, where  $\alpha = 1$ , is not explained in this analysis.

Thus, the principal's choice of the share ratio  $\alpha$  determines the agent's choice of cost-reducing activity: in particular,  $\xi_i$  decreases as  $\alpha$  increases; that is, the larger the share of costs paid for by the government, the smaller the effort expended to lower costs.

From (4) and (6) we obtain

$$k_i = k_i(\alpha) = (1 - \alpha)h'^{-1}(1 - \alpha) - h(h'^{-1}(1 - \alpha)). \quad (7)$$

The contract is awarded by means of a first-price, sealed-bid auction. Knowing the form of the contract they will be awarded if successful, the potential agents choose their bids,  $b_i$ . A symmetric Nash equilibrium of this bidding game is characterized by a bid function  $B$  such that, if all agents other than  $i$  bid  $b_j = B(c_j^*)$ , then agent  $i$  bids  $b_i = B(c_i^*)$ ; we assume that such an equilibrium exists.

Suppose all potential agents  $j$  other than  $i$  follow the bidding strategy  $b_j = B(c_j^*)$ . If  $B$  is a strictly monotonic function, then the probability that firm  $i$  is the lowest bidder is the probability that  $B(c_i^*) > b_j$  for all other firms  $j$ . Thus, the probability that firm  $i$  wins with bid  $b_i$  is  $[1 - G(B^{-1}(b_i))]^{n-1}$ . Firm  $i$ 's *ex ante* expected utility of profits is

$$EAU = [EU((1 - \alpha)(b_i - c_i^* - w) + k_i(\alpha))][1 - G(B^{-1}(b_i))]^{n-1}. \quad (8)$$

Firm  $i$  chooses  $b_i$  to maximize expected utility. If  $B$  is a Nash-equilibrium bidding strategy, then  $b_i = B(c_i^*)$ . Substituting this into the first-order condition from (8) yields

$$(1 - \alpha) \frac{EU'}{EU} B'(c_i^*) = \frac{(n-1)g(c_i^*)}{1 - G(c_i^*)} \quad \text{for all } i = 1, \dots, n, \quad (9)$$

where  $EU'$  is the expected value of  $U'$  over  $w$ .

To analyze further the agent's first-order condition (9), assume that the agents have constant absolute risk aversion; that is<sup>7</sup>

$$U(x) = \frac{1 - e^{-\lambda x}}{\lambda} \quad \text{for some } \lambda \geq 0. \quad (10)$$

*Lemma 1.* Under constant absolute risk aversion and with  $0 \leq \alpha < 1$ , the selected agent's maximum expected profit is

$$E\pi_i(c_i^*) = -\frac{1}{\lambda} \left[ \log(n-1) - (n-1) \log(1 - G(c_i^*)) - \log \left( \int_{-\infty}^{\infty} e^{\lambda(1-\alpha)wf(w)} dw \right) \right. \\ \left. + \log \int_{c_i^*}^{c_h} e^{-\lambda(1-\alpha)(c-c_i^*)} (1 - G(c))^{n-2} g(c) dc \right]. \quad (11)$$

The corresponding expected utility, conditional on being selected, is

$$EU(c_i^*) = [1 - G(c_i^*)]^{-(n-1)} e^{\lambda(1-\alpha)c_i^*} (1 - \alpha) \int_{c_i^*}^{c_h} [1 - G(c)]^{n-1} e^{-\lambda(1-\alpha)c} dc. \quad (12)$$

Moreover, the first-order condition (12) defines the global maximum *ex ante* expected utility (8).

*Proof.* See the Appendix.

<sup>7</sup> Constant absolute risk aversion is assumed because it enables the differential equation (9) to be solved (see the Appendix).

**4. The optimal linear contract**

■ We assume that the principal is risk neutral and acts as a von Stackelberg leader, designing a contract (that is, choosing  $\alpha$ ) to minimize his expected payment, while taking into account each potential agent's response (as described by Lemma 1).<sup>8</sup>

If the agent with cost  $c^*$  makes the lowest bid, the expected payment by the principal is

$$T(c^*) = E((1 - \alpha)B(c^*) + \alpha c). \tag{13}$$

Thus, the total payment by the principal will be, on average,

$$\tau = n \int_{c_l}^{c_h} T(c^*) [1 - G(c^*)]^{n-1} g(c^*) dc^*. \tag{14}$$

*Theorem 2.* If the potential agents are risk averse, the principal minimizes his expected total payment by choosing  $\alpha$  to satisfy

$$0 = \frac{\alpha}{h''(h^{-1}(1 - \alpha))} \frac{\int_{-\infty}^{\infty} w e^{\lambda(1-\alpha)w} f(w) dw}{\int_{-\infty}^{\infty} e^{\lambda(1-\alpha)w} f(w) dw} - n \int_{c_l}^{c_h} \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)} (c - c^*) [1 - G(c)]^{n-2} g(c) dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)} [1 - G(c)]^{n-2} g(c) dc} [1 - G(c^*)]^{n-1} g(c^*) dc^*. \tag{15}$$

If the potential agents are risk neutral,  $\alpha$  must satisfy

$$0 = \frac{\alpha}{h''(h^{-1}(1 - \alpha))} - n \int_{c_l}^{c_h} \int_{c^*}^{c_h} [1 - G(c)]^{n-1} dc G(c^*) dc^*. \tag{16}$$

*Proof.* From (1), (3), (4), and (13), we obtain

$$T(c^*) = E\pi_i + c^* + [h(\xi) - \xi]. \tag{17}$$

The first term in (15) comes from differentiating the terms in the brackets in (17):

$$\frac{\partial}{\partial \alpha} [h(\xi) - \xi] = (h'(\xi) - 1) \frac{\partial \xi}{\partial \alpha} = \frac{\alpha}{h''(h^{-1}(1 - \alpha))}. \tag{18}$$

The last step follows from (6).

Using (11) and noting that  $\int_{c_l}^{c_h} n [1 - G(c^*)]^{n-1} g(c^*) dc^* = 1$ , we find that the second and third terms in (15) are equal to  $\partial(E\pi_i)/\partial \alpha$ .

Since  $Ew = 0$ , the second term in (15) disappears in the risk-neutral case (that is,  $\lambda = 0$ ); thus, (15) reduces to (16). *Q.E.D.*

The three terms in (15) determining the optimal choice of  $\alpha$  correspond to the three separable effects of  $\alpha$  on the agent's actions. We shall call these the moral-hazard effect, the risk-sharing effect, and the bidding-competition effect.<sup>9</sup> The right-hand side of (15) shows

<sup>8</sup> Since from Lemma 1 the agent's first-order condition identifies the global maximum, problems associated with using the agent's first-order condition as a constraint in the principal's optimization problem (Grossman and Hart, 1983; Rogerson, 1985) do not arise here. This is so since assuming a linear contract essentially assumes away these problems.

<sup>9</sup> The first and second effects are the usual principal-agent effects: they have been analyzed with a linear payment function by Scherer (1964), Berhold (1971), Cummins (1977), and Weitzman (1980). The third effect is new to the principal-agent literature.

the rate at which the principal's expected payment decreases as  $\alpha$  increases. Thus, (15) simply equates the marginal benefit to the principal of increasing  $\alpha$  (lower expected payment owing to increased bidding competition and more risk sharing) to the marginal cost of increasing  $\alpha$  (higher expected payment owing to lower cost-reduction effort by the agent).

To interpret the moral-hazard effect, note that the net social return to the agent's cost-reduction activities is  $[\xi - h(\xi)]$ . The first term in (15) is obtained by differentiating this with respect to  $\alpha$ ; thus, it measures the marginal social gain of a change in  $\alpha$  through changing the agent's cost-controlling behavior. Because this change is a lump sum, the principal can extract all of it.

To interpret the risk-sharing effect, note that the utility cost to the agent of the random variable  $w$ , given that the realization  $w$  reduces his profit by  $(1 - \alpha)w$ , is

$$\psi(\alpha) \equiv \int_{-\infty}^{\infty} e^{\lambda(1-\alpha)w} f(w) dw. \quad (19)$$

Thus, the dollar cost to the principal of imposing this risk upon the agent (that is, the risk premium) is  $-(1/\lambda) \log \psi(\alpha)$ . Therefore, the marginal gain to the principal of reducing the agent's risk is

$$p(\alpha) \equiv \frac{\partial}{\partial \alpha} \left( -\frac{1}{\lambda} \log \psi(\alpha) \right), \quad (20)$$

which is the second term in (15). If the agents are risk neutral, then obviously this term disappears, as in (16).

The tradeoff between the moral-hazard effect and the risk-sharing effect comprises the standard principal-agent problem (Weitzman (1980), for example).<sup>10</sup> One can show that, if the bidding-competition effect is ignored, the optimal  $\alpha$  satisfies  $0 \leq \alpha < 1$ ; moreover,  $\alpha = 0$  only if the agent is risk neutral. Thus, the principal bears some of the risk unless the agent is risk neutral, and the agent always bears some of the risk.

The bidding-competition effect is the most interesting of the three components of the principal's first-order condition (15) because it is new to the principal-agent problem.<sup>11</sup> Other things being equal, bids decline as  $\alpha$  rises. For example, if  $\alpha = 0$ , the agent must cover his entire costs with the bid. This constrains his bid to be high. On the other hand, with  $\alpha$  close to 1, the agent can largely ignore his costs in making his bid. An increase in the proportion of the winning bidder's costs that are covered by the principal has an effect similar to a reduction in the variance of the distribution of expected costs among bidders and forces them to bid lower.

Thus, the bidding-competition effect works in the same direction as the risk-sharing effect and, together, these two effects trade off against the moral-hazard effect. In contrast to the standard principal-agent problem, which has a corner solution if the agent is risk neutral, in the present model there remains a tradeoff in the risk-neutral case: the moral-hazard effect must be weighed against the bidding-competition effect.

By (11) the cost to the principal of inducing competition in bidding is linear in

$$-\frac{1}{\lambda} \log \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)} [1 - G(c)]^{n-2} g(c) dc.$$

Differentiating this term with respect to  $\alpha$  yields the bidding-competition term in (15).

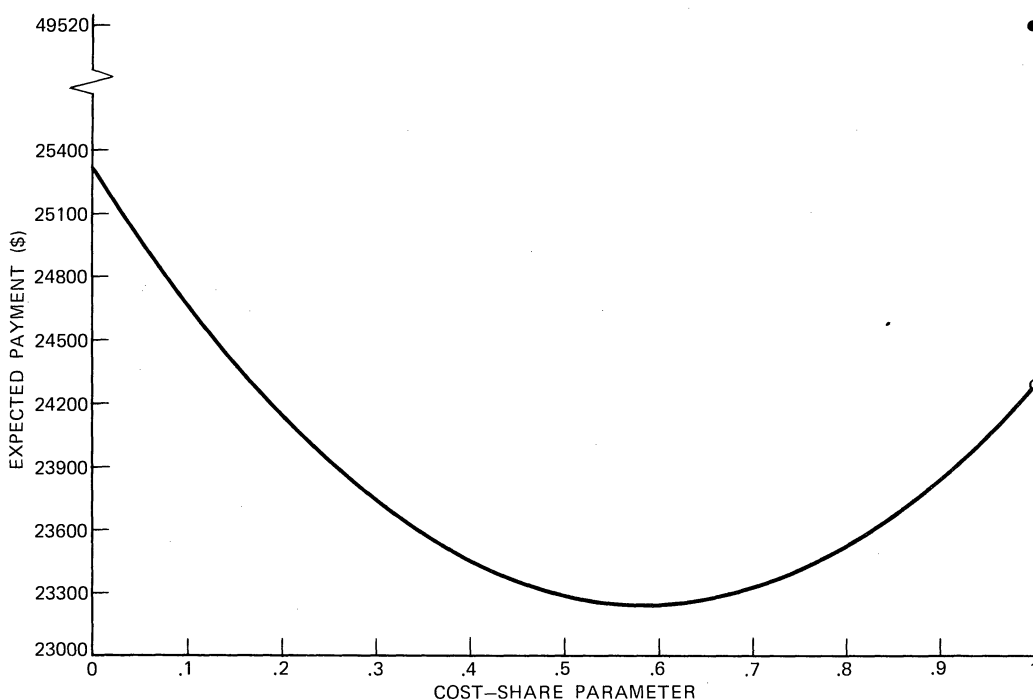
*Theorem 3.* The bidding-competition term in (15) is strictly positive for all finite numbers of potential agents  $n$ , and vanishes as  $n \rightarrow \infty$ .

*Proof.* See the Appendix.

<sup>10</sup> Note that even without the bidding-competition effect the present problem differs from the standard principal-agent problem in that the principal does not know the selected agent's expected costs; that is, in this problem there is adverse selection as well as moral hazard.

<sup>11</sup> It should be stressed that this effect exists even if the bidders are risk neutral.

FIGURE 1  
 VARIATION OF EXPECTED GOVERNMENT PAYMENT WITH COST-SHARE PARAMETER



By inspection,  $\alpha = 1$  can never be a solution of (15), so that a cost-plus contract is never optimal. According to Theorem 3, with many potential agents, the bidding-competition effect is negligible, because bidder profits are nearly zero for any  $\alpha$ . Then  $\alpha$  is determined almost entirely by the tradeoff between the risk-sharing and moral-hazard effects. Hence,  $\alpha$  tends to zero as  $n \rightarrow \infty$  if and only if the bidders are risk neutral. Otherwise,  $\alpha$  remains bounded away from zero, even when there are many bidders.<sup>12</sup>

Figure 1 presents the results of a simulation of the effects of varying the share ratio  $\alpha$ . The simulation is based on an actual fixed-price contract awarded by the Ontario Ministry of Natural Resources and assumes no risk aversion, a uniform cost distribution  $G$ , a normal disturbance  $f$ , and a quadratic moral-hazard function  $h$ : for details see McAfee and McMillan (1986b, chap. 5). The value of  $\alpha$  that minimizes the government's expected payment is .586; this results in an expected payment 13% less than for the fixed-price contract ( $\alpha = 0$ ) that the government actually used in this case. Note the discontinuity in expected payment at  $\alpha = 1$ , which results from the fact that, with  $\alpha < 1$ , the lowest-cost firm bids lowest. Thus, the government, in selecting the lowest bidder, selects the most efficient firm. With  $\alpha = 1$ , however, bids bear no relationship to expected costs, so the lowest-cost firm is probably not selected.<sup>13</sup>

<sup>12</sup> The foregoing results assumed a nondegenerate distribution  $G$  of expected costs across potential agents. If, however,  $G$  were degenerate, so that all potential agents had the same expected costs, the principal could ignore the bidding-competition effect. Thus, an alternative situation in which fixed-price contracts ( $\alpha = 0$ ) are optimal is when all firms are risk neutral and all have identical expected costs.

<sup>13</sup> The discontinuity at  $\alpha = 1$  may mean that no optimal  $\alpha$  exists (because it is possible that the solution to equation (19) puts  $\alpha$  arbitrarily close to, but not equal to, one). This is, however, of no significance for choosing the optimal contract in practice: a contract with  $\alpha$  equal to .99, say, would perform very much like a cost-plus contract in stimulating bidding competition and sharing risk, but, unlike a cost-plus contract, it would induce the bidders to reveal their relative expected costs, and so would dominate the cost-plus contract.



TABLE 1 Simulations of Optimal Incentive Contracts

Contract	Number of Bidders	Lowest Bid (\$)	Highest Bid (\$)	Saving from Optimal Incentive Contract Relative to Fixed-Price Contract		Optimal $\alpha$
				(\$)	(%)	
1	2	674,223	688,442	7,097	1.05	.25
2	4	1,085,020	1,385,220	73,909	6.81	.55
3	4	48,345	58,852	2,157	4.46	.46
6	2	152,741	158,097	4,329	2.83	.40
7	2	213,142	213,614	26	.01	.03
8	7	4,142,150	5,202,600	48,741	1.18	.24
9	3	448,520	539,469	49,052	10.94	.72
10	3	1,061,000	1,094,050	3,412	.32	.14
11	5	522,524	694,054	24,044	4.60	.45
12	4	100,596	134,706	5,791	5.33	.50
13	4	34,800	62,200	12,178	34.99	.99
14	4	1,094,750	1,391,880	72,131	6.59	.55
15	4	355,211	627,802	121,150	34.11	.99
16	5	515,408	648,013	15,546	3.02	.38
17	7	440,642	623,777	11,951	2.71	.34
18	5	1,658,600	1,768,700	4,064	.25	.12
19	4	93,571	123,736	8,284	8.85	.62
20	4	55,800	64,800	1,455	2.61	.36
21	3	194,090	263,460	52,027	26.81	.99
23	3	921,055	945,695	2,198	.24	.12

Source: McAfee and McMillan (1986b, chaps. 4, 5).

Other simulations based on Ontario Government contract data indicate that savings of about 9% can typically be achieved by using the optimal incentive contract instead of a fixed-price contract. For 60% of the contracts considered, the simulations indicated savings of more than 2% from the switch to the optimal incentive contract. Table 1 gives a typical set of simulations, based on data from fixed-price contracts for construction and maintenance projects let by the Ontario Ministry of Transportation and Communications in 1984. Note that in these simulations the bidders are assumed to be risk neutral, so that all of the savings from incentive contracts are attributable to the increased bidding competition.

## 5. A special case

■ We have thus far assumed linearity of the contract, additive separability of the cost function, and constant absolute risk aversion. For the analysis to be potentially useful in actual government-firm contracting, we must simplify the condition determining the optimal share ratio  $\alpha$  (equation (15)). We now do this by introducing special functional forms.

Suppose that the distribution of firms' costs  $G(c^*)$  is exponential:

$$G(c^*) = 1 - e^{-\mu(c^*-c)} \quad \text{for some } \mu > 0. \quad (21)$$

One can show that, if and only if costs are distributed exponentially, all firms have equal expected profits in the event of being awarded the contract.<sup>14</sup>

<sup>14</sup> If there is no finite upper bound on costs (as in this special case), there is a continuum of equilibria. But uniqueness of equilibrium can be restored by assuming that the principal has a reservation price that is finite, but high enough that the probability of receiving at least one bid is unity.

From (11) the agent's expected profit, conditional on being selected, is

$$E\pi = \frac{1}{\lambda} \log \left( \psi \left( 1 + \frac{\lambda(1-\alpha)}{\mu(n-1)} \right) \right), \quad (22)$$

and from (12) his expected utility is

$$EU = \frac{(1-\alpha)}{\lambda(1-\alpha) + (n-1)\mu}. \quad (23)$$

Thus, both expected utility and expected profit decrease in  $n$  and  $\alpha$ . The larger the number of bidders, the lower are bids and the lower are *ex post* profits. The larger the share ratio  $\alpha$ , the more competitive is the bidding and the lower are *ex post* profits. Expected utility and expected profit decrease in  $\mu$ . Since  $(1/\mu)^2$  is the variance of the cost distribution, this implies that profits increase with the variance in expected costs.<sup>15</sup> Finally, expected utility increases in the risk-aversion parameter  $\lambda$ , but the effect of  $\lambda$  on expected profits is ambiguous:

$$\frac{\partial E\pi}{\partial \lambda} \cong 0 \quad \text{as} \quad E\pi \cong (1-\alpha)p(\alpha) + \frac{EU}{\lambda^2}, \quad (24)$$

with  $EU$  given by (23). Thus, for small risks profits increase in risk aversion.

The principal expects to pay (from (13), (14), and (23)):

$$\tau = \left( c_l + \frac{1}{\mu} \right) + h(h'^{-1}(1-\alpha)) - h'^{-1}(1-\alpha) + \frac{1}{\lambda} \log \psi + \frac{1}{\lambda} \log \left[ 1 + \frac{\lambda(1-\alpha)}{\mu(n-1)} \right]. \quad (25)$$

The principal's expected payment falls as the number of bidders  $n$  increases. It increases in  $1/\mu$ : this implies that it increases with the variance in costs. It is ambiguous in the risk-aversion parameter  $\lambda$ .

If the cost disturbance  $w$  comprises many small independent shocks, the Central Limit Theorem makes the distribution  $f(w)$  approximately normal. It can be shown that, if  $f$  is normal with mean zero and variance  $\sigma^2$ , then the second term in (15), the risk-sharing term, becomes

$$p(\alpha) = \lambda(1-\alpha)\sigma^2. \quad (26)$$

We further simplify by assuming the function  $h$  to be quadratic. Let  $h'' = h_0$ . It follows from (1) and (6) that with quadratic  $h$  the difference between expected production cost under a cost-plus contract and expected production cost under a fixed-price contract is  $1/h_0$ . Thus  $1/h_0$  provides a measure of moral hazard.

Inserting all of these special assumptions, we simplify the first-order condition (15) to a quadratic equation in  $\alpha$ :

$$0 = \frac{\alpha}{h_0} - \lambda(1-\alpha)\sigma^2 - \frac{1}{\lambda(1-\alpha) + \mu(n-1)}. \quad (27)$$

Denote the right-hand side of (27) by  $M(\alpha)$ . Recall that a fixed-price contract ( $\alpha = 0$ ) is optimal if and only if both  $\lambda = 0$  and  $n = \infty$ . If either of these conditions fails, an incentive contract ( $0 < \alpha < 1$ ) is prescribed; substitution into (27) shows that then  $M(0) < 0$ . If there is only one root for (27) in  $(0, 1)$ , it must be the case that at the optimal  $\alpha$ ,  $M'(\alpha) > 0$ . To see this, note that  $M(\alpha)$  is the difference between the marginal benefits and marginal costs to the principal of increasing  $\alpha$ ; if  $M'(\alpha) < 0$  when  $M(\alpha) = 0$ , the principal could lower his expected payment by further increasing  $\alpha$ . Now the effects on the optimal share ratio  $\alpha$  of

<sup>15</sup> Note that, since the mean of the exponential distribution is  $c_l + (1/\mu)$ , the standard deviation,  $1/\mu$ , can be varied independently of the mean. Hence the derivative of profits with respect to  $1/\mu$  measures the rate at which profits change with the standard deviation of costs.

changes in the exogenous parameters can be predicted by using the Implicit Function Theorem and  $M'(\alpha) > 0$ :

- (1)  $\partial\alpha/\partial\sigma^2 > 0$ : increasing the variance of the cost disturbance increases the optimal value of  $\alpha$ . The greater is this variance, the more risk-averse agents must be sheltered from risk.
- (2)  $\partial\alpha/\partial(1/\mu) > 0$ : increasing the variance of potential agents' expected costs increases the optimal  $\alpha$ . The larger is this variance, the weaker is the competition in bidding, so that  $\alpha$  must be raised in compensation.
- (3)  $\partial\alpha/\partial n < 0$ : the more firms that enter the bidding, the smaller is the optimal  $\alpha$ . The greater the bidding competition, the more the principal is able to use  $\alpha$  to offset moral hazard.
- (4)  $\partial\alpha/\partial h_0 > 0$ : the more markedly do returns to cost-reduction effort diminish, the larger is the optimal  $\alpha$ . As noted above,  $1/h_0$  measures the extent of moral hazard, and so the larger is  $h_0$ , the less the principal needs to use  $\alpha$  to provide incentives for cost reduction.
- (5)  $\partial\alpha/\partial\lambda \cong 0$ : changes in the agent's risk aversion have an ambiguous effect on the optimal  $\alpha$ . The more risk averse is the selected agent, the greater the need for risk sharing. This tends to make  $\alpha$  large. But the more risk averse are the potential agents, the lower they will bid to make more likely their winning the contract; thus the principal need not make  $\alpha$  large to stimulate bidding competition.

Suppose the government imposes a sales tax on inputs used by the agent or a tax on the income of the agent's employees. Then the agent's expected cost is  $(1 + T)c^*$  with the tax  $T$  (instead of  $c^*$  without the tax). The new cost distribution is, instead of (21),

$$G(c) = 1 - e^{-[\mu/(1+T)](c-(1+T)c)}, \quad (28)$$

The effect of the tax is to increase the standard deviation of expected costs (which is now  $(1 + T)/\mu$ ). From (22) the agent's expected profit rises; and from (25) the principal's expected payment rises by more than the increase in expected costs. This is so since the increase in the variance of expected costs weakens competition at the bidding stage. The surprising implication is that, if the government taxes a firm with which it deals, the resulting increase in government expenditure exceeds the tax revenue. A tentative policy recommendation is that taxes on inputs used by contractors in government projects should be rebated.

## 6. Summary and conclusions

■ This article combines a bidding model with a principal-agent model. The two aspects of the analysis interact in an important way. The optimal contract trades off, as in the usual principal-agent analysis, moral hazard against risk sharing. But the bidding process adds a new element to this tradeoff. The terms of the contract affect how the firms bid in the initial competition for the contract. This bidding-competition effect serves to reinforce the risk-sharing effect in determining the optimal contract. Even if the bidders are risk neutral, so that there is no need for risk sharing, there is still a tradeoff between stimulating competition in the initial bidding and giving the winning firm incentives to limit its costs.<sup>16</sup>

Most of the theoretical literature on auctions presumes that each bidder's true valuation of an item is known only to the bidder himself, so that the payment can depend only upon the amounts bid. If valuations are observable *ex post*, however, the payment can be made contingent upon both the bids and the true valuation. For example, in auctions of oil rights

<sup>16</sup> Care should be exercised in drawing policy conclusions from this analysis, as what is assumed away might in practice be significant. The principal is assumed to be able to observe the contractor's realized cost without incurring auditing costs. The observed cost, presumably in practice accounting cost, must be a close approximation to opportunity cost. The principal does not incur any costs in evaluating bids from prospective contractors, who similarly have no bid-preparation costs. The bidders differ only in their expected costs, which are independently distributed.

on government-owned land, payments equal the amount bid plus a royalty based on the amount of oil extracted. Publishing rights for books are sometimes auctioned, with payments to the author depending both on the bid and, via a royalty, on the book's sales. The general lesson from this article is that an optimally designed auction will, where feasible, make payment depend upon true valuation as well as on bids. Since it may not be possible to monitor the winning bidder's subsequent actions, making payment depend upon valuation introduces moral-hazard problems. Thus, the gains from making payment depend upon valuation must be weighed against the losses from moral hazard.

## Appendix

■ The proofs of Lemma 1 and Theorem 3 follow.

*Proof of Lemma 1.* With the utility function (10) we have

$$EU(x - (1 - \alpha)w) = \frac{1}{\lambda} \left[ 1 - e^{-\lambda x} \int_{-\infty}^{\infty} e^{\lambda(1-\alpha)w} f(w) dw \right] = \frac{1}{\lambda} [1 - \psi(\alpha)e^{-\lambda x}] \quad (A1)$$

and

$$EU'(x - (1 - \alpha)w) = (1 - \lambda EU(x - (1 - \alpha)w)). \quad (A2)$$

Thus, from (9) and (A2) we obtain

$$\frac{dEU}{dc^*} = EU'(\beta B'(c^*) - (1 - \alpha)) = \frac{(n-1)g(c^*)}{1 - G(c^*)} EU - (EU')(1 - \alpha) = \theta'(c^*)EU - (1 - \alpha), \quad (A3)$$

where

$$\theta(c^*) = -(n-1) \log(1 - G(c^*)) + \lambda(1 - \alpha)c^*. \quad (A4)$$

Equation (A3) is a linear ordinary differential equation. The solution to (A3) is for some constant  $K$

$$EU = [1 - G(c^*)]^{-(n-1)} e^{\lambda(1-\alpha)c^*} \left[ K - (1 - \alpha) \int_{c_1}^{c^*} [1 - G(c)]^{n-1} e^{-\lambda(1-\alpha)c} dc \right]. \quad (A5)$$

Thus, by (8) we have

$$EAU = e^{\lambda(1-\alpha)c^*} \left[ K - (1 - \alpha) \int_{c_1}^{c^*} [1 - G(c)]^{n-1} e^{-\lambda(1-\alpha)c} dc \right]. \quad (A6)$$

But (10) also implies that

$$EAU|_{c=c_h} = 0.$$

Thus, from (A6) we get

$$K = (1 - \alpha) \int_{c_1}^{c_h} [1 - G(c)]^{n-1} e^{-\lambda(1-\alpha)c} dc.$$

Hence, as required, (12) follows from (A5).

Rearranging (12), we obtain

$$EU = \frac{1}{\lambda} [1 - [1 - G(c^*)]^{-(n-1)} \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)} (n-1) [1 - G(c)]^{n-2} g(c) dc]. \quad (A7)$$

From (3), (10), and (A7), we have

$$e^{-\lambda(\beta B(c^*) - (1-\alpha)c^* + k)} = \frac{[1 - G(c^*)]^{-(n-1)}}{\Psi(\alpha)} \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)} (n-1) [1 - G(c)]^{n-2} g(c) dc. \quad (A8)$$

Using (A8) and (10) yields, as required, (11).

It remains to show that (12) provides the unique local maximum to (8). The uniqueness follows from (A3). To check that the solution is a maximum, consider a firm with cost  $c^*$  that proposes to bid  $b_0 = B(c_0^*)$ . Then, by (8) and (A8), we get

$$\begin{aligned} EAU &= \frac{1}{\lambda} [1 - G(c_0^*)]^{n-1} \left[ 1 - \int e^{-\lambda(k + \beta B(c_0^*) - (1-\alpha)(c^* + w))} f(w) dw \right] = \frac{1}{\lambda} [1 - G(c_0^*)]^{n-1} [1 - e^{-\lambda(1-\alpha)\lambda(c_0^* - c^*)} \Psi e^{-\lambda(k + \beta B(c_0^*) - (1-\alpha)c_0^*)}] \\ &= \frac{1}{\lambda} [1 - G(c_0^*)]^{n-1} [1 - e^{-\lambda(1-\alpha)\lambda(c_0^* - c^*)} (n-1) [1 - G(c_0^*)]^{-(n-1)} \int_{c_0^*}^{c_h} e^{-\lambda(1-\alpha)(c-c_0^*)} [1 - G(c)]^{n-2} g(c) dc] \\ &= \frac{1}{\lambda} [1 - G(c_0^*)]^{n-1} - e^{\lambda(1-\alpha)c^*} \int_{c_0^*}^{c_h} e^{-\lambda(1-\alpha)c} [1 - G(c)]^{n-2} g(c) dc. \end{aligned}$$

Thus, we have

$$\frac{dEAU}{dc_0^*} = \frac{1}{\lambda}(n-1)[1 - G(c_0^*)]^{n-2}g(c_0^*)[-1 + e^{\lambda(1-\alpha)(c^* - c_0^*)}]$$

$$= 0 \quad \text{if and only if} \quad c_0^* = c^*$$

$$\left. \frac{d^2EAU}{dc_0^{*2}} \right|_{c_0^* = c^*} = (n-1)[1 - G(c_0^*)]^{n-2}g(c_0^*)(-(1-\alpha))e^{\lambda(1-\alpha)(c^* - c_0^*)} < 0.$$

Hence, a strict local maximum has been found, which, by continuity and the fact that the first-order conditions have a unique solution, is a global maximum. *Q.E.D.*

*Proof of Theorem 3.* Define

$$q(\alpha) \equiv \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)}(c-c^*)[1 - G(c)]^{n-2}g(c)dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)(c-c^*)}[1 - G(c)]^{n-2}g(c)dc}$$

$$= \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}c \frac{(n-1)[1 - G(c)]^{n-2}}{[1 - G(c^*)]^{n-1}}g(c)dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c} \frac{(n-1)[1 - G(c)]^{n-2}}{[1 - G(c^*)]^{n-1}}g(c)c}$$

$$\equiv \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}cr_n(c)dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc} - c^*.$$

By (15) it is sufficient to show that  $q(\alpha) > 0$  for all finite  $n$  and  $\lim_{n \rightarrow \infty} (\sup_{\alpha} q(\alpha)) = 0$ . Note that  $r_n$  is a probability density function, so that

$$\int_{c^*}^{c_h} r_n(c)dc = 1.$$

Define

$$\mu_n = \int_{c^*}^{c_h} cr_n(c)dc$$

and

$$\sigma_n^2 = \int_{c^*}^{c_h} (c - \mu_n)^2 r_n(c)dc.$$

Note that

$$\mu_n = c \frac{[1 - G(c)]^{n-1}}{[1 - G(c^*)]^{n-1}} \Big|_{c^*}^{c_h} + \int_{c^*}^{c_h} \left[ \frac{1 - G(c)}{1 - G(c^*)} \right]^{n-1} dc = c^* + \int_{c^*}^{c_h} \left[ \frac{1 - G(c)}{1 - G(c^*)} \right]^{n-1} dc.$$

Thus,

$$\lim_{n \rightarrow \infty} \mu_n - c^* = \lim_{n \rightarrow \infty} \int_{c^*}^{c_h} \left[ \frac{1 - G(c)}{1 - G(c^*)} \right]^{n-1} dc = 0, \tag{A9}$$

if, for all  $c > c^*$ ,  $G(c) < G(c^*)$ . To prove (A9) note that  $g(c) > 0$  implies  $\exists \delta > 0$  such that

$$[1 - G(c)]/[1 - G(c^*)] < 1 - \delta \text{ if } c > (c^* + \epsilon/2).$$

Thus,

$$\int_{c^*}^{c_h} \left[ \frac{1 - G(c)}{1 - G(c^*)} \right]^{n-1} dc < \int_{c^*}^{c^* + (\epsilon/2)} dc + \int_{c^* + (\epsilon/2)}^{c_h} (1 - \delta)^{n-1} dc = \frac{\epsilon}{2} + (1 - \delta)^{n-1}.$$

Thus, by choosing  $n$  large enough, we can make  $(1 - \delta)^{n-1} < \epsilon/2$ , and hence  $\mu_n - c \rightarrow 0$ . Now note that

$$q(\alpha) = \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}(c - \mu_n)r_n(c)dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc} + \mu_n - c^* \geq 0.$$

Because  $e^{-\lambda(1-\alpha)c}$  is convex in  $c$  and

$$\int_{c^*}^{c_h} (1 + \mu_n - c)r_n(c)dc = 1,$$

we have

$$\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc \leq \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c^*}r_n(c)dc = e^{-\lambda(1-\alpha)c^*}$$

and

$$\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}(1 + \mu_n - c)r_n(c)dc \geq \exp -\lambda(1-\alpha) \int_{c^*}^{c_h} c(1 + \mu_n - c)r_n(c)dc = e^{-\lambda(1-\alpha)\mu_n - \sigma^2}.$$

Thus,

$$\begin{aligned} \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}(c - \mu_n)r_n(c)dc &\leq \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc - e^{-\lambda(1-\alpha)\mu_n - \sigma^2} \\ &\leq \int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc - e^{-\lambda(1-\alpha)\mu_n}. \end{aligned}$$

Therefore,

$$\begin{aligned} q(\alpha) &= \frac{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}(c - \mu_n)r_n(c)dc}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc} + \mu_n - c^* \leq 1 - \frac{e^{-\lambda(1-\alpha)\mu_n}}{\int_{c^*}^{c_h} e^{-\lambda(1-\alpha)c}r_n(c)dc} + \mu_n - c^* \leq 1 - \frac{e^{-\lambda(1-\alpha)\mu_n}}{e^{-\lambda(1-\alpha)c^*}} + \mu_n - c^* \\ &= 1 - e^{-\lambda(1-\alpha)(\mu_n - c^*)} + (\mu_n - c^*). \end{aligned}$$

Thus,  $q(\alpha) \rightarrow 0$  if there is no interval  $[c_0, c_1] \subseteq (c_l, c_h)$  for which  $g(c) = 0$ ,  $c_0 \leq c \leq c_1$ ,  $c_0 < c_1$ . By the definition of  $c_l \equiv \inf \{c | g(c) > 0\}$ ,  $q(\alpha)|_{c_l} \rightarrow 0$ . Therefore, the bidding-competition term, *BCE*, satisfies

$$BCE = \int_{c_l}^{c_h} q(\alpha)|_c n(1 - G(c))^{n-1} g(c)dc \rightarrow 0.$$

To see this, note that  $q(\alpha)$  is bounded for all  $c$  and  $n$ . Call this number  $M$ . Thus,

$$\begin{aligned} BCE &\leq \int_{c_l}^{c_l+\delta} q(\alpha)n(1 - G(c))^{n-1} g(c)dc + \int_{c_l+\delta}^{c_h} Mn(1 - G(c))^{n-1} g(c)dc \\ &= \int_{c_l}^{c_l+\delta} q(\alpha)n(1 - G(c))^{n-1} g(c)dc + (1 - G(c_l + \delta))^n M \\ &\leq \delta q(\alpha)|_{c_l+\delta} + (1 - G(c))^n M, \quad 0 \leq \delta \leq 1. \end{aligned}$$

For large  $n$  we can make both terms not exceed  $\epsilon/2$ . Thus  $BCE \rightarrow 0$ . *Q.E.D.*

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