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HORIZONTAL MERGERS AND ANTITRUST POLICY*

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The welfare implications of horizontal mergers are examined in the context of the Cournot-Nash model of Perry and Porter. Horizontal mergers are more likely to be welfare enhancing the more concentrated is the ownership of the non-merging firms. Mergers that create a new largest firm, or increase the size of the largest firm, reduce welfare.

I. INTRODUCTION

PERRY AND PORTER [1985] proposed an ingenious model of horizontal mergers based on a fixed industry capital stock. Their purpose was to counter the observation of Salant, Switzer, and Reynolds [1983] that, in the Cournot model where each firm has the same constant average cost, mergers tend to be unprofitable. As noted by Perry and Porter, models in which firms have constant average costs, possibly varying among firms, do not yield sensible descriptions of mergers.

In a model where firms have different but constant average costs, the merger of two firms leads to the shutdown of the high-cost firm. Such shutdowns are almost never observed in real mergers. The sole gain to the low-cost firm from the merger is the elimination of the high-cost firm as a Cournot rival. Since the low-cost firm has no capacity constraints, it has no use for the assets of the high-cost firm.

Intuitively, a merged firm should be "bigger" than either of the two pre-merger firms because it combines the assets of the two firms. Such an asset combination does occur in the quadratic-cost model of Perry and Porter and in our extension of that model.

II. THE MODEL

Our model is a special case of Perry and Porter [1985] with Cournot-Nash conjectures and no fixed costs (see also Schwartz and Baumann [1988] and Farrell and Shapiro [1990]). Firms are Cournot players with firm i, $i \in \{1, \ldots, n\}$, choosing quantity q_i given the other firms' quantities. Total output is the sum of the q_is and is denoted Q. Demand is linear, with price $p = a - bQ$. Marginal costs are also linear, with total cost of firm i equal to

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$q_i^2/2k_i$, where k_i is the firm's capital investment. Firm i chooses q_i to maximize profits

(1) $$\pi_i = (a - bQ)q_i - q_i^2/2k_i$$

Since π_i is strictly concave in q_i, the choice of q_i is given by the first order condition

(2) $$p = a - bQ = (b + k_i^{-1})q_i$$

Define

(3) $$\beta_i = \frac{bk_i}{bk_i + 1}$$

and

(4) $$B = \sum_{i=1}^{n} \beta_i$$

Then the following results are easily derived[1]

(5) $$Q = \frac{a}{b}\left(\frac{B}{1+B}\right)$$

(6) $$p = \frac{a}{1+B}$$

(7) $$q_i = \frac{a}{b}\left(\frac{\beta_i}{1+B}\right)$$

Let s_i be firm i's market share. Then

(8) $$s_i \equiv q_i/Q = \beta_i/B$$

Let $E = -\frac{P}{Q}\frac{\partial Q}{\partial P}$ be the elasticity of demand. Then in equilibrium

(9) $$B = 1/E$$

Finally, welfare (consumer surplus plus profits) is given by:

(10) $$W = \frac{a^2}{2b}\left(\frac{B}{1+B} + \left(\frac{B}{1+B}\right)^2 h\right) = \frac{a^2}{2b}\left(\frac{1}{1+E} + \frac{h}{(1+E)^2}\right)$$

where $h = \sum_{i=1}^{n} s_i^2$ is the Herfindahl index. It is easily shown that, holding the number of firms fixed, W is maximized by a symmetric allocation of capital among firms.

The equilibrium value of marginal cost is a decreasing function of market

[1] All derivations are provided in a technical appendix available upon request.

share. Thus, in an asymmetric industry, with different capital levels, production is carried out inefficiently since marginal costs are not equated. This provides scope for a merger to increase welfare, since it can lead to more efficient production by making the industry "more symmetric." However, mergers will diminish industry output, thereby reducing consumer surplus. This tradeoff forms the basis for the welfare analysis executed below.

A merger is viewed as bringing the capital of two firms under a single authority. We assume, without loss of generality, that firms 1 and 2 merge. The subscript m denotes characteristics of the merged firm (e.g. $k_m = k_1 + k_2$ and s_m is the market share in equilibrium of the merged firm) or of the post-merger market (e.g. Q_m is the post-merger equilibrium industry output). The merged firm may also be viewed as a multiplant firm, operating the two former firms as "plants."[2] To see this, note that a multiplant firm operates both plants efficiently, so the marginal cost of production mc satisfies

(11) $$mc = q_1/k_1 = q_2/k_2$$

from which we obtain

(12) $$mc = \frac{q_1+q_2}{k_1+k_2}$$

For firm i, $3 \leqslant i \leqslant n$, the level β_i depends only on k_i and b (from (3)) and is therefore unaffected by the merger. The β value for the merged firm satisfies

(13) $$\max\{\beta_1, \beta_2\} < \beta_m = \frac{\beta_1+\beta_2-2\beta_1\beta_2}{1-\beta_1\beta_2} < \beta_1+\beta_2$$

Thus the value of B, which determines total industry output, changes after the merger to

(14) $$B_m = B+\beta_m-\beta_1-\beta_2 < B$$

Moreover,

(15) $$\max\{s_1, s_2\} < s_m < s_1+s_2$$

By (5) and (14), the merger causes total output and consumer surplus to fall. All non-merging firms expand production, by (7), but the merged firm contracts sufficiently to overwhelm this expansion. The merged firm, however, produces more than either of its component firms did,[3] and its market share does not fall below the market share of its largest component firm, in contrast to Salant, Switzer and Reynolds [1983] or Clarke [1987].

To characterize the welfare effects of a merger, it is useful to introduce the following notation concerning the pre-merger market:

(16) $$s = s_1+s_2, \quad s \in [0, 1]$$

[2] Welfare effects, of course, depend on the behavior of other firms as well.
[3] $\max\{q_1, q_2\} < q_m < q_1+q_2$.

(17) $z = s_1 s_2 / s^2, \quad z \in [0, 1/4]$

(18) $h_c = \sum_{i=3}^{n} s_i^2/(1-s)^2, \quad h_c \in [0,1]$

If firms 1 and 2 were to merge, the Herfindahl of the non-merging firms, i.e. the Herfindahl "conditional" on the merger, is h_c. The conditional Herfindahl is useful because it does not change after the merger occurs, unlike the Herfindahl used in antitrust enforcement.

The expression for whether a merger is welfare enhancing can be reduced to a critical value on s. That is, there is a function $s^*(h_c, E, z)$.[4] This critical value of s is increasing in h_c and appears to be indeterminate in z and E.[5] Thus, the more concentrated are the non-merging firms, the more likely is the merger to be welfare enhancing.[6] Moreover,

(19) $E \geqslant 2/3 \Rightarrow s^*(h_c, E, z) \leqslant h_c/(h_c+1)$

Equation (19) expresses a remarkable claim. If the demand is at least moderately elastic ($E \geqslant 2/3$), no merger should be allowed that will create a new largest firm. To see this, note that at least one of the firms $3 \leqslant i \leqslant n$ has share $s_i \geqslant (1-s)h_c$,[7] and (19) is equivalent to $s^* \leqslant (1-s^*)h_c$. This means for a merger to be welfare enhancing, there must be a non-merging firm whose market share exceeds the sum of the pre-merger shares of the merging firms. That is, welfare-enhancing mergers do not increase the market share of the largest firm. In particular, the largest firm is never permitted to merge. The requirement of $E \geqslant 2/3$ is necessary in the sense that, for E slightly less than 2/3, z near zero, and h_c near 1, the results fail.

An equivalent expression to (19) is[8]

(20) $s^* < h_c(1-s^*)^2 + s^{*2}$

The right hand side is the post-merger Herfindahl, as referenced by the US Department of Justice Merger Guidelines [1984]. Thus another expression for this necessary condition for welfare enhancement is the sum of the

[4] s^* is defined by the equation $0 = BS[(1+B)(1-B^2s^2z) - B^2s^2z(2-Bs)]\,[(1+B)(1-2Bsz) - 2Bsz(2-Bs)] + (2-Bs)B^2(1-s^2)h_c[2(1+B)(1-B^2s^2z) - B^2s^2z(2-Bs)] - Bs(2-Bs)(1+B(1-s))\,[(1+B)(1-2Bsz) + (1+B)(1-B^2s^2z) - B^2s^2z(2-Bs)]$.

[5] A computer program is available from the authors that calculates the welfare effects of mergers in the model. The authors have calculated the welfare effects of all mergers for which $1/2 \leqslant E \leqslant 20$; these results are also available upon request.

[6] Farrell and Shapiro [1990] also make this point. Farrell and Shapiro show that if $s < h_c/E$, then a merger will be welfare enhancing. This condition is not, however, necessary.

[7] Suppose the contrary: all firms $i = 3, \ldots, n$ have share $s_i < (1-s)h_c$. Then

$$h_c = \sum_{i=3}^{n} \left(\frac{s_i}{1-s}\right)^2 < \sum_{i=3}^{n} h_c \frac{s_i}{1-s} = h_c,$$

a contradiction.

[8] Note that (19) coincides with $s^* < h_c(1-s^*)$. Equation (20) results from multiplying both sides by $(1-s^*)$. We thank a referee for this observation.

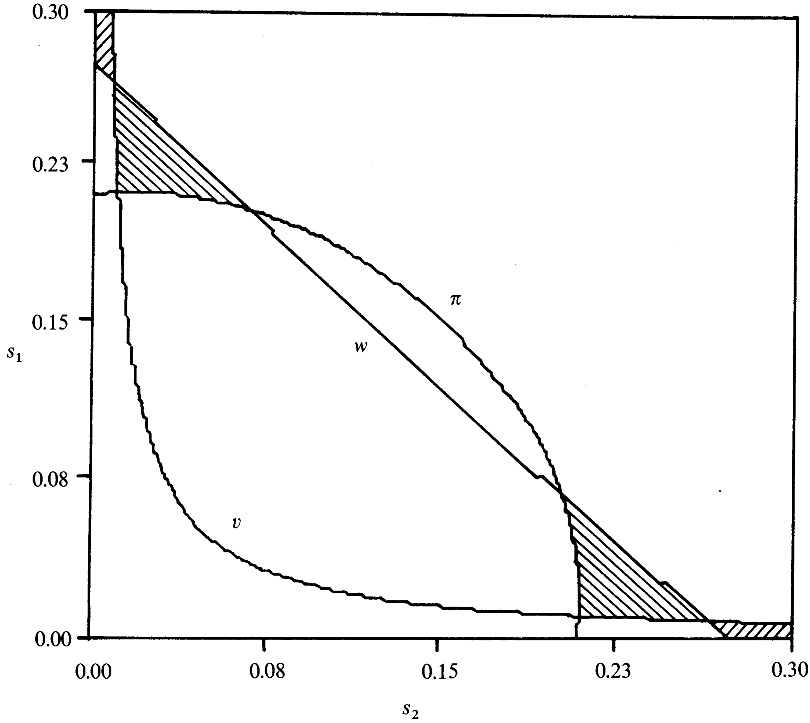

Figure 1
The market has three pre-merger firms, and the (absolute) demand elasticity is 2.0. The figure shows how the welfare effects of a merger between firms 1 and 2 depend on the market shares, s_1 and s_2, of the two firms. A merger between firms 1 and 2 is profitable if $s_1 + s_2$ is greater than the π curve. A merger between firms 1 and 2 violates the Merger Guidelines if $s_1 + s_2$ is greater than the V curve. A merger between firms 1 and 2 is welfare enhancing if $s_1 + s_2$ is less than the W curve. The area ///// represents profitable mergers that reduce welfare but do not violate the Merger Guidelines. The area \\\\\ represents profitable mergers that enhance welfare but do violate the Merger Guidelines.

merging firms' shares does not exceed the magnitude of the post-merger Herfindahl.

In Figure 1, we illustrate the effects of mergers as the merging firms' shares change, for E equal to 2 and h_c equal to 1; i.e. there are three pre-merger firms. As Figure 1 illustrates, the criterion for whether a merger is welfare enhancing depends primarily on $s = s_1 + s_2$ and hardly at all on $z = s_1 s_2 / s^2$, because the welfare line (marked W) is nearly straight. Over the range of values taken on by E, h_c, and z, the variation in s^* as z changes is less than 0.02, with this value taken only when demand is inelastic ($E \leqslant 0.8$). That is to say, a decision rule that ignores the disparity in the merging firms' market shares does remarkably well. This diagram illustrates that the Merger Guidelines can permit profitable, welfare-reducing mergers (marked by /////) and prohibit profitable, welfare-enhancing mergers (marked by \\\\\).

V. CONCLUSIONS

We have developed an equilibrium model of the welfare effects of horizontal mergers. The model has a number of testable predictions because the parameters can be recovered from observables (price, elasticity of demand, and firm market share). These predictions can be used to test the plausibility of the model's assumptions, by examining whether they held for past mergers. We list some predictions that arise in a post-merger environment:

(i) Industry output decreases.
(ii) The output and market share of all non-merging firms increase.
(iii) The merged firm produces less than the sum of the component firms' pre-merger outputs; thus its market share is less than the sum of the pre-merger market shares of the component firms.
(iv) The merged firm produces more than either of the component firms produced prior to the merger. Also, its market share is larger than either of the component firms' market shares.
(v) The "conditional Herfindahl"—i.e. the Herfindahl of the non-merging firms—is unchanged by the merger. Furthermore, the ratio of the market shares of any two non-merging firms is unchanged by the merger.

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