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Multiproduct Equilibrium Price Dispersion*

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The model of Burdett and Judd (*Econometrica* **51** (1993), 955–969) is generalized to the case of many goods. Consumers choose the best price observed for each good. There are two classes of equilibria, those that involve constant expected profits for each good independently of price and those with increasing profits for each good in price. A continuum of the latter type always exists. These equilibria are qualitatively different than the unique equilibrium of the single-good case. *Journal of Economic Literature* Classification Numbers: D83, D43, L13.

INTRODUCTION

The texbook "Law of One Price" is empirically irrelevant. Prices of homogeneous goods, such as brand-name soft drinks, paper towels, and canned goods, vary significantly from store to store and from week to week. The profession has proposed a variety of models that account for random prices using equilibrium models (for an excellent survey, see McMillan and Rothschild [10]), but each model focuses on the market for a single good. This paper, in contrast, considers a model with a randomized price equilibrium over many goods and proves conclusions qualitatively different than that obtained in a single good market.

In a single-good market, Burdett and Judd [2] propose a clever model of equilibrium price dispersion, which generates a unique price dispersion in an environment with identical firms and consumers. The critical assumption that Burdett and Judd make is that some consumers observe exactly one price, while others see more than one price.[1] That this generates a price dispersion is easily observed. If some firms offer price p with positive probability, it pays to undercut p slightly, so as to sell to those consumers receiving two or more prices with minimum p. Such competition cannot force the price to marginal cost, since it is more profitable to price higher

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[1] See the closely related analysis of Robert and Stahl [12], Stahl [15], Varian [16], and the seminal model of Butters [4].

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and sell only to those consumers receiving one price offer. Thus, an equilibrium price dispersion results.

For many categories of commodities, consumers purchase more than one good, and firms advertise prices of several goods. If consumers only purchase at one store, consumers will be exclusively concerned with the price of their commodity bundle (the sum of the prices), and the multiproduct problem degenerates to a single-good problem, as considered by Burdett and Judd. If, in contrast, consumers can costlessly visit all the stores from which they receive advertisements, a store will sell good i to a consumer only if it offered the lowest price on that good, from among all the prices on good i the consumer received. This case, which is qualitatively different than the single-good problem, is examined here.

Both cases represent extremes of a more reasonable problem, where it is both costly to advertise and costly to visit stores. Free store visits are reasonable, however, when the consumer does not physically visit the store, but intead phones, orders via a catalog, or watches television "infomercials." Most of the search costs will arise from obtaining the catalog price, and actually ordering the item will be relatively inexpensive.

The predictions in the multiproduct case differ qualitatively from the single-good case. The equilibrium need not be unique. As a function of the price of the ith good, profits need not be invariant to price, as must occur with a single good, to permit randomization. This occurs since raising the ith price, which induces strictly higher profits on the ith good, may require lowering another price, with associated lower profits, leaving the sum of profits constant. Raising the ith price alone, in this circumstance, would induce consumers to search further, lowering profits below those available when consumers do not search further.

Burdett and Judd actually consider several related models, which differ according to how the number of price offers that consumers receive is generated. Only one of these models will be considered here, wherein a consumer who pays a search cost c receives n price offers with probability α_n. If no offer is acceptable, the consumer may sample again from the same distribution at the same cost c, with recall of previous offers.[2] Each offer is a tuple $(p_1, ..., p_m)$, where p_i is the price offered on the ith good. Each consumer will purchase all m commodities. Search occurs with recall.

To provide a concrete example, consider a consumer searching for several major appliances, e.g., a washer and dryer, or receiver, CD player, and speakers. At cost c, they may visit a retail store, which carries a number of brands and carries n brands with probability α_n. Consumers buy

[2] The buyer's side of the market has been analyzed, for the case $\alpha_1 = 1$, by Burdett and Malueg [3]. Carlson and McAfee [7] consider buyer optimization in the multiproduct, no recall case.

the lowest priced brand for each of the goods (i.e., there are no compatibility issues). A consumer might visit a retailer such as Sears in search of a washer and dryer. Depending on the prices observed, the consumer might purchase a Kenmore washer and a Maytag dryer. If the lowest prices are too high, the consumer can visit another retail outlet and obtain a new sample of prices, by incurring the cost c again. Once the consumer has decided on his purchases, he phones in orders to the retail outlet or outlets that carry the goods with the lowest prices observed (that is, the consumer need not incur the cost c again). In equilibrium, the consumer will visit at most one outlet, but I allow for repeated sampling.

A second example is provided by buying services. There are several companies that offer consumer buying services, such as Citibank. A member can call one of these services and be quoted prices on a large variety of consumer goods, and receive a quote that represents the best prices that the buying service could obtain from its suppliers. A call to a second buying service would produce a similar best price from a potentially different group of suppliers. Neither of these examples fit precisely with the model, because of the assumed zero recall cost and the assumption that a second expenditure of the search cost produces an independent and identically distributed sample of prices. However, the model does seem to be a logical starting point for an analysis of multiproduct price dispersion.

The next section sets out the model and proves some preliminary lemmas. The third section analyzes one class of equilibria, that corresponds closely to the single-good case. However, these equilibria need not exist, and conditions sufficient for existence are provided. An exact characterization of when these equilibria exist is available for the case of two goods. The fourth section investigates a different class of equilibria. It turns out that a continuum of these equilibria always exist. The final section provides conclusions and a critique of the theory.

THE MODEL

Each firm offers m goods and chooses a price vector $(p_1, ..., p_m)$ for the goods. There is a continuum of firms and consumers. Each consumer, upon paying the search cost c, learns the vector of prices of n firms with exogenous probability α_n. The consumer desires to minimize the sum of the purchase prices for the m goods, minus the search costs, and takes the joint distribution of prices at each firm as given. The equilibrium concept is perfect Bayesian. Firms simultaneously set prices, which will involve randomization, given beliefs about consumers' search rules. Consumers search optimally, to minimize expected cost, given their beliefs about firms' pricing rules, which are correct in equilibrium.

I assume that

$$\alpha_0 + \alpha_1 < 1, \tag{1}$$

and,

$$\alpha_1 > 0. \tag{2}$$

Inequality (1) rules out the monopoly price as an equilibrium, while (2) rules out marginal cost as an equilibrium pricing strategy. Assume that the number of offers received is a finite integer with probability one

$$\sum_{n=0}^{\infty} \alpha_n = 1, \tag{3}$$

and the expected number of offers is also finite,

$$\bar{n} \equiv \sum_{n=0}^{\infty} n\alpha_n < \infty.$$

Firms logically expect to have more competitors, conditioned on a consumer receiving their offer, than the consumers expect to have offers. Intuitively, there are n ways a given firm's price offer can be present when the consumer receives n offers. The probability that a firm has $n-1$ competitors, given that a consumer receives the firm's offer, is

$$\gamma_n = n\alpha_n/\bar{n}. \tag{4}$$

A formal proof of (4) is provided in McAfee and McMillan [8, Lemma 1], but a simple argument or intuition exists. Consider asking parents (consumers) how many children (price offers) they have, and suppose the proportion of parents reporting n is α_n. Now consider asking children (firms) how many children (price offers) their parents have, and one sees that, for each parent with n children, there are n reports of n, and thus the proportion of reports of n is $n\alpha_n/\bar{n}$, where $\bar{n}$ is the expected number of children.

Suppose that consumers enjoy zero-transportation costs, so that the lowest observed price on any good is the price the consumer pays. Let $F_i(f_i)$ be the marginal distribution (density) of the price of the ith good (only symmetric equilibria will be considered, so F_i does not depend on the firm's identity), and note that the distribution of prices can contain no interior mass points. For suppose the ith distribution has a mass point at p. A slight reduction in price then brings a discrete jump in the probability that this price is accepted, and consequent increase in profits, contradicting the optimality of the mass point. Since $\alpha_1 > 0$, no firm charges marginal cost.

The consumer's acceptance set is defined by the set of prices such that the sampling cost exceeds the expected gain from an additional draw. Let

$G_i(g_i)$ be the marginal distribution (density) of the lowest price a consumer receives from any one sample. Then

$$1 - G_i(p_i) = \sum_{n=0}^{\infty} \alpha_n (1 - F_i(p_i))^n. \tag{5}$$

If the support of f_i is $[L_i, H_i]$, then

$$G_i(L_i) = 0, \tag{6}$$

$$G_i(H_i) = 1 - \alpha_0. \tag{7}$$

The acceptance set of the consumer is the set of prices $(p_1, ..., p_m)$ satisfying, by (6),

$$c \geqslant \sum_{i=1}^{m} \int_{L_i}^{p_i} (p_i - q)\, g_i(q)\, dq = \sum_{i=1}^{m} \int_{L_i}^{p_i} G_i(q)\, dq. \tag{8}$$

Independence of the price distribution is not being assumed in Eq. (8), which holds because expectation is a linear operator. Firms that price inside the acceptance set enjoy profits on the ith good of (recall (4))

$$\pi_i(p_i) = (p_i - k_i) \sum_{n=1}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(p_i))^{n-1}, \tag{9}$$

for each consumer receiving the firm's offer, where k_i is the marginal cost of the ith good. No firm will ever price below k_i, because increasing this price would reduce the loss on the ith good.[3] Since a decrease in price leaves a price vector inside the acceptance set, π_i must be nondecreasing in price over $p_i \in [L_i, H_i]$.

No firm will ever price outside the acceptance set. To see this fact, consider a vector of prices $\mathbf{p} = (p_1, ..., p_m)$ that lies outside the acceptance set. Let θ_i be the probability that a consumer receiving such an offer purchases the ith good, conditional on not receiving a better offer on good i in the first round of offers. Since $\mathbf{p}$ was not in the acceptance set, at least one $\theta_i < 1$. Let $\hat{p}_i$ be the consumer's expected price on good i, conditional on receiving the offer $\mathbf{p}$. By the definition of θ_i,

$$\hat{p}_i = \theta_i p_i + (1 - \theta_i)\, E\{P_i \mid P_i < p_i\},$$

where the expectation is taken over the expected price, conditional on not accepting p_i. The vector of prices $(\hat{p}_1, ..., \hat{p}_m)$ lies within the consumer's acceptance set, since it is defined as the expected value of the consumer's accepted prices. We immediately have $k_i \leqslant \hat{p}_i \leqslant p_i$. Moreover, a firm

[3] Should a firm price below k_i, raising that price to k_i and lowering another price by an equal amount, will be strictly profitable.

choosing $(\hat{p}_1, ..., \hat{p}_m)$ makes a strictly higher profit than by choosing $\mathbf{p}$, since the profits associated with $\mathbf{p}$ are

$$\begin{aligned}
&\sum_{i=1}^{m} \theta_i(p_i - k_i) \sum_{n=1}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(p_i))^{n-1} \\
&\quad < \sum_{i=1}^{m} [\theta_i(p_i - k_i) + (1 - \theta_i)(E\{P_i \mid P_i \leqslant p_i\} - k_i)] \\
&\quad\quad \times \sum_{n=1}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(p_i))^{n-1} \\
&\quad = \sum_{i=1}^{m} (\hat{p}_i - k_i) \sum_{n=1}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(p_i))^{n-1} \\
&\quad \leqslant \sum_{i=1}^{m} (\hat{p}_i - k_i) \sum_{n=1}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(\hat{p}_i))^{n-1},
\end{aligned}$$

which is the profit associated with charging $(\hat{p}_1, ..., \hat{p}_m)$.

Thus, for π_i to be strictly increasing at p_i, the price offer must be on the outer border of the acceptance set. Let A be the set of acceptable prices, defined by Eq. (8).

LEMMA 1. *A is convex.*

All proofs are relegated to the Appendix.

Two classes of equilibria will be investigated. In the first, π_i is constant for all i. Such equilibria are qualitatively like the equilibria in the one-good case. Generally, in these equilibria, offered prices are in the interior of the acceptance set A. I call these equilibria *constant profits equilibria.* In the second class of equilibria, offered prices fall on the border of the acceptance set, and the profit on any one good is increasing in the offered price. What constrains a firm from raising prices is that this would prompt further search, in the event that the consumer receives no other offer. In such an equilibrium, which I call *frontier*, obviously the sum of prices is constant, to permit randomization. There may be other, hybrid cases of these equilibria, which are not investigated here.

CONSTANT PROFITS EQUILIBRIA

Consider first equilibria with π_i constant on $[L_i, H_i]$. From (9),

$$\begin{aligned}
\pi_i &= L_i - k_i \\
&= (H_i - k_i) \frac{\alpha_1}{\bar{n}}. \qquad (10)
\end{aligned}$$

Define r_i by

$$c = \int_{L_i}^{r_i} G_i(p_i)\, dp_i. \tag{11}$$

The maximum acceptable price that can be charged on any one commodity is r_i.

LEMMA 2. *In a constant profits equilibrium,* $r_i = H_i$.

From Lemma 2,

$$\begin{aligned} c &= \int_{L_i}^{H_i} (H_i - q)\, g_i(q)\, dq && \text{(by (11))} \\ &= (H_i - k_i)\, G_i(H_i) \\ &\quad - \int_{L_i}^{H_i} (q - k_i) \sum_{n=1}^{\infty} n\alpha_n (1 - F_i(q))^{n-1} f_i(q)\, dq && \text{(by (5) and (6))} \\ &= (H_i - k_i)(1 - \alpha_0) - \int_{L_i}^{H_i} \bar{n}\pi_i f_i(q)\, dq && \text{(by (7) and (9))} \\ &= (H_i - k_i)(1 - \alpha_0 - \alpha_1) && \text{(by (10))}. \end{aligned} \tag{12}$$

Thus, using (10) and (12),

$$H_i = k_i + \frac{c}{1 - \alpha_0 - \alpha_1}, \tag{13}$$

$$L_i = k_i + \frac{\alpha_1}{\bar{n}} \frac{c}{1 - \alpha_0 - a_1}, \tag{14}$$

and,

$$\pi_i = \frac{\alpha_1 c}{\bar{n}(1 - \alpha_0 - \alpha_1)}. \tag{15}$$

The value of $F_i(p_i)$ is given by (9). It is easily seen that there is a unique solution to (9) for $F_i(p_i)$, since the right-hand side of (9) is a decreasing function of argument $F_i(p_i)$. Because π_i is independent of i, I suppress the subscript for this section. Note that (9) and (13)–(15) completely characterize a constant profits equilibrium, provided it exists. The equilibrium exists provided the set of offered prices falls within the acceptance set A.

Note that firms will not price outside of the acceptance set, and that, by construction, profits are maximized within the acceptance set, since they are constant. Thus, to procure an equilibrium, I must show that there is a distribution function F, which has marginal distribution functions equal to the F_i given by (9) and the support of F falling in the acceptance set. It will prove useful to introduce the notation

$$\varphi(x) \equiv \sum_{n=0}^{s} \alpha_n x^n, \tag{16}$$

and,

$$\beta(x) = \frac{1-\varphi(x)}{\varphi'(x)}. \tag{17}$$

Note that, for $x \in [0, 1]$,

$$\beta'(x) \leqslant -1. \tag{18}$$

It is useful to transform the prices and work directly with the probability distribution. Thus define

$$y_i = 1 - F_i(p_i). \tag{19}$$

Observe that p_i has distribution F_i if and only if y_i is uniformly distributed. By (5),

$$G_i(p_i) = 1 - \sum_{n=0}^{s} \alpha_n y_i^n = 1 - \varphi(y_i). \tag{20}$$

The utility of the notation φ is apparent from Eq. (20), which expresses the price distribution facing consumers in terms of φ. It is less obvious why the transformation (19) simplifies the analysis so dramatically. The utility of (19) arises at least partially because the central problem of equilibrium construction is to prove that there exists a joint distribution of prices with given marginals and given support. Equation (19) transforms the problem so that the given marginals are uniform. The function β arises naturally, if not intuitively, in the analysis, as the following lemma shows.

LEMMA 3. *$p \in A$ if and only if*

$$\sum_{i=1}^{m} (\beta(y_i) + y_i) \leqslant \beta(0) + m - 1. \tag{21}$$

Let A^* be the set of acceptable prices in terms of y_i, that is,

$$A^* = \left\{ (y_1, ..., y_m) \in [0, 1]^m \;\middle|\; \sum_{i=1}^{m} (\beta(y_i) + y_i) \leqslant \beta(0) + m - 1 \right\}. \tag{22}$$

Remark 1. A constant profits equilibrium exists if and only if there is a distribution over $\mathbf{y} = (y_1, ..., y_m)$ with uniform marginal distributions of y_i and support in A^*. It may prove helpful to bear in mind that high prices correspond to low values of y_i by (19), so that A^* is an increasing set.

Lemma 3 transforms the question of existence of a constant profits equilibrium into a question about the existence of a distribution over A^* with uniform marginal distributions. The following two lemmas give necessary and sufficient conditions for such an equilibrium distribution to exist.

LEMMA 4. *If a constant profits equilibrium exists, then*

$$A^* \subseteq \left\{ (y_1, ..., y_m) \in [0, 1]^m \;\middle|\; \sum_{i=1}^{m} y_i \geqslant m-1 \right\}. \tag{23}$$

LEMMA 5. *If*

$$A^* \supseteq \left\{ (y_1, ..., y_m) \in [0, 1]^m \;\middle|\; \sum_{i=1}^{m} y_i^{1/(m-1)} \geqslant m-1 \right\}, \tag{24}$$

then a constant profits equilibrium exists.

Remark 2. The conditions (23) and (24) of Lemmas 4 and 5 coincide for $m=2$, which leads to the following theorem.

THEOREM 6. *Suppose $m=2$. A constant profits equilibrium exists if and only if*

$$(\forall x \in [0, 1]) \quad \beta(x) + \beta(1-x) \leqslant \beta(0).$$

Moreover, if β is convex, then a constant profits equilibrium exists.

Remark 3. If $\alpha_0 + \alpha_1 + \alpha_2 = 1$, then β is convex, and a constant profits equilibrium exists for $m=2$.

Remark 4. A straightforward example of this model is the binomial case, where

$$\alpha_n = \binom{N}{n} a^n (1-a)^{N-n},$$

for $n \leqslant N$, and $\alpha_n = 0$ otherwise. Each of the N firms are represented at only some of the retail outlets, and firm i is represented at a given retail outlet with probability a, independently of whether the other firms are represented at this outlet. In this case, $\varphi(x) = (ax + 1 - a)^N$, and β is easily seen to be convex. In the limiting case of $a = b/N$, and $N \to \infty$, $\alpha_n = e^{-b}(b^n/n!)$, the support of the two prices, for $m=2$, is a rectangular hyperbola.

That is, a constant profits equilibrium exists with the property that $(p_1 - k_1)(p_2 - k_2)$ is constant for all offered prices (p_1, p_2).

Remark 5. For $m = 2$, no constant profits equilibrium exists if

$$\sum_{n=2}^{\infty} \alpha_n[\tfrac{1}{2}\alpha_1 - 2^{-n}(\alpha_1 + n(1 - \alpha_0))] > 0.$$

This condition is satisfied, for instance, if $\alpha_1 = \alpha_5 = \frac{1}{2}$, with $\alpha_n = 0$ for $n \neq 1, 5$.

Our final result of this section examines only the convex β case, and thus applies to the cases given in Remarks 3 and 4.

THEOREM 7. *Suppose β is convex. Then a constant profits equilibrium exists if and only if*

$$\beta\left(\frac{1}{m}\right) \leqslant \frac{1}{m}\beta(0) + \frac{m-2}{m}. \tag{25}$$

The condition (25) *must fail for m sufficiently large, and in particular, fails if*

$$m \geqslant 1 + \frac{2(1 - \alpha_0)\alpha_2}{\alpha_1(1 - \alpha_0 - \alpha_1)}.$$

For sufficiently large numbers of goods, there is no equilibrium where profits are constant on each good separately. As is shown below, however, there are always a plethora of equilibria where profits are not constant on each good separately, but only the sum of the profits is constant.

Remark 6. For $m = 2$, there are often many different constant profits equilibrium distributions, although all equilibria have the same marginal distributions. In general, the dimensionality of the support of prices may be anything from 1 to m. The proof of Theorem 7 gives dimensionality of the support at 1, the proof of Lemma 5 gives dimensionality $m - 1$, and an example is given in the Appendix with dimensionality m. In this case, a consumer will receive two offers, one with higher prices on every single good, with positive probability. Moreover, in this example, the lowest price on each good offered by any firm is in the support of the prices distribution.[4]

One interesting feature of the constant profits equilibrium, from (15), is that the profits on each good are the same as the profits in the one-good

[4] If a firm offers the highest price in the support, then it must offer the lowest price on all other goods, by Lemma 2. In the example, however, the vector of the lowest prices $(L_1, \ldots, L_m)$ is in the support of prices.

case and are independent of the number of goods. In a constant profits equilibrium, then, the firms' profits behave as if the consumers were independently searching for the goods, rather than jointly searching. The distribution of firm prices, however, will typically be negatively correlated, although the marginal distribution of any one good's price is the same as if it were determined in a single-good environment.

I now turn to the case of equilibria where offered prices fall on the border of the acceptance set.

FRONTIER EQUILIBRIA

By a frontier equilibrium, I mean an equilibrium where each price vector offered by a firm is on the border of A, so that the firm is discouraged from raising any given price alone, because this would induce further search by consumers in the event that the consumer received only that firm's offer. As before, let $y_i = 1 - F_i(p_i)$, and employ a superscript * on functions when they are defined in terms of y_i instead of p_i. Thus, by (16), (17), and (19),

$$\begin{aligned} \pi_i^*(y_i) = \pi_i(p_i) &= (p_i - k_i) \sum_{n=0}^{\infty} \frac{n\alpha_n}{\bar{n}} (1 - F_i(p_i))^{n-1} \\ &= (F_i^{-1}(1 - y_i) - k_i)\, \varphi'(y_i)/\bar{n}. \end{aligned} \tag{26}$$

The analog to Lemma 3 for this case, which holds for any equilibrium, is:

LEMMA 8.

$$A^* = \left\{ (y_1, ..., y_m) \in [0, 1]^m \,\middle|\, \sum_{i=1}^{m} \left(\pi_i^*(y_i)\, \beta(y_i) - \int_{y_i}^{1} \pi_i^*(s)\, ds \right) \leqslant c/\bar{n} \right\}.$$

A frontier equilibrium has the following properties. First, the π_i functions are nondecreasing, and thus π_i^* is nonincreasing, so that a firm does not wish to lower price (increase y_i). Second, all offered prices are on the border of A. Finally, the sum of profits for prices on the border of A is constant, so that firms are willing to randomize over this space. The following theorem shows that at least a continuum of such solutions exist.

THEOREM 9. *There exists a family of frontier equilibria, indexed by* $\lambda \in (0, 1/\beta(0))$, *satisfying*

$$\pi_i^*(y_i) = \pi^*(y_i) = \frac{c\, e^{\int_0^{y_i} [\lambda(1 + \beta'(s))/(1 - \lambda\beta(s))]\, ds}}{\bar{n}(\beta(0) - \int_0^1 e^{\int_0^t [\lambda(1 + \beta'(s))/(1 - \lambda\beta(s))]\, ds}\, dt)}, \tag{27}$$

with

$$p_i = k_i + \frac{\bar{n}\pi^*(y_i)}{\varphi'(y_i)}.$$

In this family, if $F_i(p_i) = 1$, then $F_j(p_j) = 0$ for all $j \neq i$. Moreover, total expected profits in any of these equilibria are less than total profits in any constant profits equilibrium, if one exists.

Theorem 9 shows that the multiproduct case is qualitatively different than the single-good case, becaue many equilibria exist. Moreover, it shows that a constant profits equilibrium, if one exists, involves higher profits, and thus higher cost to consumers, than *any* frontier equilibrium. This result is paradoxical, because consumers' costs are lower when they are pushed to the limit of their willingness to pay in all realizations (bear in mind that a frontier equilibrium involves pricing on the border of the acceptance set; having received only one offer, a consumer is indifferent between accepting it and searching again, while this is typically not true of a constant profits equilibrium). Of course, the dilemma is resolved by noting that the distribution of prices is not held constant in this comparison. In a frontier solution, consumers are at their limit of their willingness to pay beause the price distribution is advantageously lower, encouraging search in the event of high prices, and therefore depressing profits.

In these equilibria, a firm which prices at the highest possible price on good i (i.e., $y_i = 0$) will price at the lowest possible price on all other goods. This property is a feature of any equilibrium where the entire border of the acceptance set is randomized over (subject to price exceeding marginal cost), such as those constructed in the proof of Theorem 9. However, there may exist other frontier equilibria where firms do not randomize over all of the acceptance set, and these equilibria may have different profits.

Note that the minimum profits that a firm could possibly get in any equilibrium is $c\alpha_1/(n(1-\alpha_0))$, because the firm has no competition with probability $\gamma_1 = \alpha_1/\bar{n}$, conditional on a consumer receiving the firm's offer, and the expected cost to the consumer of obtaining another offer is $c/(1-\alpha_0)$. Thus, even if all other firms charged marginal cost, no consumer would search again after receiving an offer of $\sum_{i=1}^{m} k_i + c/(1-\alpha_0)$, which yields expected profits of $c\alpha_1/(\bar{n}(1-\alpha_0))$, in the event that all other firms charge marginal cost. As $\lambda \to 1/\beta(0)$, the sum of profits converge to this amount, the theoretical minimum. This level of profits may actually be less than the profits in the one-good case; that is, begin with a one-good situation, introduce a second good, and the sum of profits may fall. The condition for this possibility is that the probability of a consumer receiving one offer, α_1, exceeds the probability of competing offers, $\sum_{n=2}^{r} \alpha_n$.

CONCLUSION

When Rothschild [13] called for a model of an equilibrium price dispersion, the profession responded enthusiastically with a plethora of models. These models fell into two broad categories: those that depended on exogenous differences in firms' production costs and consumers' search costs,[5] and those, like Butters [4], that produced price dispersion with identical firms and consumers, by firms' randomization. This paper shows that uniqueness of symmetric equilibrium in the latter category is not robust to an increase in the number of goods. The multiple equilibria differ from each other in substantial ways; in particular, firm profits and consumer costs differ across equilibria. Thus, a major advantage of the price dispersion model of Burdett and Judd [2], which is a tractable, unique equilibrium, is not robust to the introduction of multiple goods.

Although this paper offers closed forms for many equilibria, these formulae are not much help for using the model to summarize data on a multiple-good price dispersion, which would be useful for empirical studies like Carlson and Gieseke [5] because of the multiple equilibria of the model. One trivial testable prediction of the model is that, in any equilibrium, no firm will offer the highest price on more than one good. In most of the equilibria, prices across goods will tend to be negatively correlated. This is qualitatively similar to loss leaders, in that firms with high prices on some goods will tend to offer lower prices on others. However, firms will not price below marginal cost in the present model, which seems to be a feature of loss leaders at grocery stores.

An interesting multiproduct competition problem concerns the desirability of bundling in competition.[6] Would a firm desire to offer a bundle of commodities at a single price, rather than, or in addition to, pricing the components separately? A resolution of the indeterminacy of equilibria in the pricing subgame is necessary to address this question.

None of the equilibrium price dispersion papers account for consumers learning about the distribution of prices, which seems to me to be an integral part of price dispersions in most markets.[7] Moreover, I expect that

[5] See the references in Carlson and McAfee [6], which is an example of this category.

[6] Adams and Yellen [1] and McAfee *et al.* [9] have considered bundling by monopolists.

[7] It is not surprising that the literature has avoided learning about the distribution in an equilibrium price dispersion context, for it appears difficult to even provide conditions under which the set of acceptable prices, for a fixed history of observed prices, is an interval. Thus, it appears difficult to guarantee that the quantity demanded is nonincreasing in price. See Rothschild [14] and Morgan [11]. The difficulty is that, when a consumer observes a low price, the consumer concludes that the distribution is favorable, thus encouraging further search.

entry of firms into markets with price dispersions is an important factor in determining the equilibrium, and this should be explicitly accounted for in the model. Finally, equilibrium price dispersions provide an excellent model of imperfect competition, preferable in many applications to Cournot competition models, and thereby should be used to analyze traditional industrial organization topics, such as advertising, research and development, and merger.

Appendix

Proof of Lemma 1. Consider any two price vectors **p** and **q** in the acceptance set. Let $\lambda \in [0, 1]$, and let $\mathbf{z} = \lambda\mathbf{p} + (1-\lambda)\mathbf{q}$. Define

$$\xi(\lambda) = \sum_{i=1}^{m} \int_{L_i}^{\lambda p_i + (1-\lambda) q_i} G_i(p_i)\, dp_i.$$

Note that **p**, **q** are in A if and only if $\xi(0)$, $\xi(1)$ are less than c. A is convex if this, in turn, implies that $\xi(\lambda)$ is less than c. But

$$\xi''(\lambda) = \sum_{i=1}^{m} g_i(z_i)(p_i - q_i)^2 \geqslant 0,$$

so $\xi(\lambda) \leqslant \lambda\xi(0) + (1-\lambda)\,\xi(1) \leqslant c$, as desired. ∎

Proof of Lemma 2. Since all charged prices are acceptable, $r_i \geqslant H_i$. Now suppose that $r_i > H_i$. Then consider a firm which prices at L_j for $j \neq i$, and at r_i. By (8), this is within the acceptance set. The firm earns π_j on goods $j \neq i$, and $(r_i - k_i)\,\alpha_1/\bar{n} > \pi_i$ on good i, increasing total profit, a contradiction. ∎

Proof of Lemma 3. First, note that

$$\sum_{i=1}^{m} \int_{L_i}^{p_i} (p_i - q)\, g_i(q)\, dq$$

$$= \sum_{i=1}^{m} \left[(p_i - k_i)\, G_i(p_i) - \int_{L_i}^{p_i} (q - k_i) \sum_{n=1}^{\infty} n\alpha_n (1 - F_i(q))^{n-1} f_i(q)\, dq \right]$$

(by (5))

$$= \sum_{i=1}^{m} \left[(p_i - k_i)\, G_i(p_i) - \int_{L_i}^{p_i} \bar{n}\pi f_i(q)\, dq \right]$$

(by (9))

$$= \sum_{i=1}^{m} [(p_i - k_i)\, G_i(p_i) - \bar{n}\pi F_i(p_i)]$$

$$= \sum_{i=1}^{m} [(p_i - k_i)\, G_i(p_i) + \bar{n}\pi(1 - F_i(p_i))] - m\bar{n}\pi$$

$$= \sum_{i=1}^{m} \left[\frac{\bar{n}\pi}{\varphi'(y_i)} (1 - \varphi(y_i)) + \bar{n}\pi y_i \right] - m\bar{n}\pi$$

(by (9), (16), (19), and (20))

$$= \bar{n}\pi \left[\sum_{i=1}^{m} (\beta(y_i) + y_i) - m \right].$$

(by (17))

By (8) and (15), then, $\mathbf{p} \in A$ if and only if

$$\bar{n}\pi \frac{1 - \alpha_0 - \alpha_1}{\alpha_1} = c \geqslant \bar{n}\pi \left[\sum_{i=1}^{m} (\beta(y_i) + y_i) - m \right]$$

Noting that $\beta(0) = (1 - \alpha_0)/\alpha_1$ yields (21). ∎

Proof of Lemma 4. Suppose there exists an equilibrium distribution, and let $\mathbf{z}$ be on the border of A^*. Define $M = \{1, ..., m\}$ and, for $I \subseteq M$, $A_I = \{\mathbf{x} \in A^* \backslash \{\mathbf{z}\} \mid i \in I$ if and only if $x_i \leqslant z_i\}$. Recall that A^* is an increasing set, by (19). Note that

$$A_M = \varnothing \qquad \text{(since } \mathbf{z} \text{ is on the border of } A^*\text{)} \qquad \text{(A1)}$$

$$A^* \backslash \{\mathbf{z}\} = \bigcup_{I \neq M} A_I \qquad \text{(from (A1) and definition of } A_I\text{)} \qquad \text{(A2)}$$

$$I \neq J \Rightarrow A_I \cap A_J = \varnothing. \qquad \text{(from the definition of } A_I\text{)} \qquad \text{(A3)}$$

$$\bigcup_{\{I \mid i \in I\}} A_I = \{\mathbf{x} \in A^* \backslash \{\mathbf{z}\} \mid x_i \leqslant z_i\} \qquad \text{(from the definition of } A_I\text{).} \qquad \text{(A4)}$$

Finally, since y_i is $\mathbf{U}[0, 1]$, and there are no mass points, so the probability of $\mathbf{z}$ is zero, by (A4) we have

$$Pr(\mathbf{y} \in \bigcup_{\{I \neq M \mid i \in I\}} A_I) = z_i. \qquad \text{(A5)}$$

Therefore,

$$\sum_{i \in M} z_i = \sum_{i \in M} Pr\left(\mathbf{y} \in \bigcup_{\{I \neq M \mid i \in I\}} A_I\right)$$

$$= \sum_{i \in M} \sum_{\{I \neq M \mid i \in I\}} Pr(\mathbf{y} \in A_I) = \sum_{I \neq M} \sum_{i \in I} Pr(\mathbf{y} \in A_I)$$

$$= \sum_{I \neq M} |I| \, Pr(\mathbf{y} \in A_I) \leqslant \sum_{I \neq M} (m-1) \, Pr(\mathbf{y} \in A_I) = m - 1.$$

The first step is from (A5), the second is from (A3), the third is a rearrangement of sums, the fourth holds since there are $|I|$ elements in I, the fifth is from (A1), and the last step is from (A2). ▮

Proof of Lemma 5. Let $S = \{\mathbf{x} \in [0, 1]^m \mid \sum_{i=1}^m x_i = 1\}$.

Consider the uniform distribution over S, and let $\mathbf{X}$ denote this random variable. Note that

$$Pr(X_i \geqslant x_i) = (1 - x_i)^{m-1}.$$

To see this, let

$$z_j = \begin{cases} X_j & j \neq i \\ X_i - x_i & j = i \end{cases},$$

and note that $Z = \{\mathbf{X} \in S \mid X_i \geqslant x_i\} = \{\mathbf{z} \mid z_j \geqslant 0 \text{ and } \sum_{j=1}^m z_j = 1 - x_i\}$. That is, Z is a simplex with sum equal to $1 - x_i$. Thus it contains $(1 - x_i)^{m-1}$ of the volume of S. Since $1 - (1 - X_i)^{m-1}$ is the distribution function of X_i, it follows that $Y_i = (1 - X_i)^{m-1}$ is $\mathbf{U}[0, 1]$. Since $X \in S$, the random variable Y is defined over the set

$$\left\{\mathbf{Y} \in [0, 1]^m \,\middle|\, \sum_{i=1}^m (1 - Y_i^{1/(m-1)}) = 1\right\}$$

$$= \left\{\mathbf{Y} \in [0, 1]^m \,\middle|\, \sum_{i=1}^m Y_i^{1/(m-1)} = m - 1\right\}.$$

If this set is in A^*, we have constructed an equilibrium. ▮

Proof of Theorem 6. For $m = 2$, the necessary and sufficient conditions of Lemmas 4 and 5 coincide. Both require that, for $y_1 = x \in [0, 1]$, $y_2 = 1 - y_1$, $\beta(x) + x + \beta(1 - x) + 1 - x \leqslant \beta(0) + 1$, or,

$$\beta(x) + \beta(1 - x) \leqslant \beta(0). \tag{A6}$$

Now suppose β is convex. Then $\beta(x)+\beta(1-x)$ is also convex, and hence maximized at an endpoint, $x \in \{0, 1\}$. But this means that (A6) holds for all x, since $\beta(1)=0$. ∎

Justification for Remark 5. Note that $2\beta(\frac{1}{2})>\beta(0)$ if and only if

$$2\frac{1-\sum_{n=0}^{\infty}\alpha_n 2^{-n}}{\sum_{n=1}^{\infty} n\alpha_n 2^{-(n-1)}}>\frac{1-\alpha_0}{\alpha_1}$$

$$\text{iff}\quad \alpha_1\left(1-\sum_{n=0}^{\infty}\alpha_n 2^{-n}\right)>(1-\alpha_0)\sum_{n=1}^{\infty} n\alpha_n 2^{-n}$$

$$\text{iff}\quad a_1(1-\alpha_0-\tfrac{1}{2}\alpha_1)-\tfrac{1}{2}(1-\alpha_0)\alpha_1>\sum_{n=2}^{\infty}\alpha_n 2^{-n}(\alpha_1+n(1-\alpha_0))$$

$$\text{iff}\quad \tfrac{1}{2}\alpha_1(1-\alpha_0-\alpha_1)>\sum_{n=2}^{\infty}\alpha_n 2^{-n}(\alpha_1+n(1-\alpha_0))$$

$$\text{iff}\quad \sum_{n=2}^{\infty}\alpha_n(\tfrac{1}{2}\alpha_1-2^{-n}(\alpha_1+n(1-\alpha_0)))>0.$$

If $\alpha_1=\alpha_5=\frac{1}{2}$, the left-hand side evaluates to $\frac{5}{64}$.

Proof of Theorem 7. (Necessity) Let $y_i=1/m$. By Lemma 4, $\mathbf{y}\in A^*$. By (21), $m[(1/m)+\beta(1/m)]\leqslant\beta(0)+m-1$, which yields (25).

(Sufficiency) Consider the following random variable $\mathbf{X}$. Choose i from $\{1, ..., m\}$ with equal probability, and then choose δ from $\mathbf{U}[0, \frac{1}{2}]$. Let $\mathbf{X}=(1-\delta, ..., 1-\delta, \delta, 1-\delta, ..., 1-\delta)$, with the δ in the ith component. Since

$$Pr(X_i\leqslant x_i)=\begin{cases}\dfrac{2x_i}{m} & x_i<\dfrac{1}{2}\\[2ex] \dfrac{1}{m}+\dfrac{m-1}{m}2\left(x_i-\dfrac{1}{2}\right) & x_i\geqslant\dfrac{1}{2},\end{cases}$$

the variable Y_i given by

$$Y_i=\begin{cases}\dfrac{2x_i}{m} & x_i<\dfrac{1}{2}\\[2ex] \dfrac{1}{m}+\dfrac{m-1}{m}(2x_i-1) & x_i\geqslant\dfrac{1}{2},\end{cases}$$

is U[0, 1]. By construction,

$$\sum_{i=1}^{m} (\beta(y_i) + y_i)$$

$$= \beta\left(\frac{2\delta}{m}\right) + \frac{2\delta}{m} + (m-1)$$

$$\times \left[\beta\left(\frac{1}{m} + \frac{m-1}{m}(2(1-\delta)-1)\right) + \frac{1}{m} + \frac{m-1}{m}(2(1-\delta)-1) \right]$$

$$= \beta\left(\frac{2\delta}{m}\right) + \frac{2\delta}{m} + (m-1)\left[\beta\left(1 - 2\frac{m-1}{m}\delta\right) + 1 - 2\frac{m-1}{m}\delta \right]$$

$$= \beta\left(\frac{2\delta}{m}\right) + (m-1)\,\beta\left(1 - 2\frac{m-1}{m}\delta\right) - \frac{2\delta}{m}((m-1)^2 - 1) + m - 1$$

$$= \beta\left(\frac{2\delta}{m}\right) + (m-1)\,\beta\left(1 - 2\frac{m-1}{m}\delta\right) - 2\delta(m-2) + m - 1.$$

Since β is convex, this expression is maximized at an endpoint, $\delta \in \{0, \frac{1}{2}\}$. At $\delta = 0$, we obtain $\beta(0) + m - 1$, which satisfies (21). At $\delta = \frac{1}{2}$, (21) yields (25). As $m \to \infty$, (25) becomes

$$\frac{1-\alpha_0}{\alpha_1} = \beta(0) \leqslant 1,$$

which is false, by (1). Finally, since β is convex,

$$\beta\left(\frac{1}{m}\right) \geqslant \beta(0) + \frac{1}{m}\beta'(0) = \frac{1-\alpha_0}{\alpha_1} - \frac{1}{m}\left(1 + \frac{(1-\alpha_0)\,2\alpha_2}{\alpha_1^2}\right).$$

Thus, a sufficient condition for (25) to fail is

$$\frac{1-\alpha_0}{\alpha_1} - \frac{1}{m}\left(1 + \frac{(1-\alpha_0)\,2\alpha_2}{\alpha_1^2}\right) \geqslant \frac{1}{m}\frac{1-\alpha_0}{\alpha_1} + \frac{m-2}{m},$$

or,

$$m \geqslant 1 + \frac{(1-\alpha_0)\,2\alpha_2}{(1-\alpha_0-\alpha_1)\alpha_1},$$

as desired. ▮

Justification for Remark 6. Let $m = 2$, and $\alpha_0 = \alpha_2 = \frac{1}{4}$, and $\alpha_1 = \frac{1}{2}$. This is the duopoly binomial case. Let (X_1, X_2) be uniformly distributed over

$\{(x_1, x_2) \mid x_i \geqslant 0,\ x_1 + x_2 \leqslant 1\}$. Define $Y_i = (1 - X_i)^2$, and note that Y_i is U[0, 1]. Moreover, $Y_2 = (1 - X_2)^2 \geqslant X_1^2$, and thus

$$\beta(Y_1) + Y_1 + \beta(Y_2) + Y_2 \leqslant \beta((1 - X_1)^2) + (1 - X_1)^2 + \beta(X_1^2) + X_1^2.$$

This yields a constant profits equilibrium if, for all $x \in [0, 1]$,

$$\beta(x^2) + \beta((1 - x)^2) + x^2 + (1 - x)^2 \leqslant \beta(0) + 1,$$

or

$$\beta(x^2) + \beta((1 - x)^2) \leqslant \beta(0) + 2x(1 - x). \tag{A7}$$

Noting that $\varphi(x) = \frac{1}{4}(1 + x)^2$, we observe that $\beta(x) = 2/(1 + x) - \frac{1}{2}(1 + x)$. We can then reexpress (A7):

$$\frac{2}{1 + x^2} - \frac{1}{2}(1 + x^2) + \frac{2}{1 + (1 - x)^2} - \frac{1}{2}(1 + (1 - x)^2) \leqslant 2 - \frac{1}{2} + 2x(1 - x). \tag{A8}$$

Tedious algebra reduces (A8) to

$$0 \leqslant x^2 - x^3 - 2x^4 + 3x^5 - x^6 = x^2(1 - x)^2(1 + x - x^2)$$

which is obviously true for $x \in [0, 1]$, and thus we have constructed a constant profits equilibrium with full dimensionality of the support of prices.

Proof of Lemma 8.

$$\sum_{i=1}^{m} \int_{L_i}^{p_i} (p_i - q)\, g_i(q)\, dq$$

$$= \sum_{i=1}^{m} \left[(p_i - k_i)\, G_i(p_i) - \int_{L_i}^{p_i} (q - k_i) \sum_{n=1}^{\infty} n\alpha_n (1 - F_i(q))^{n-1} f_i(q)\, dq \right]$$

(by (5) and (9))

$$= \sum_{i=1}^{m} \left[\frac{\bar{n}\pi_i(p_i)}{\varphi'(y_i)} (1 - \varphi(y_i)) - \int_{L_i}^{p_i} \bar{n}\pi_i(q)\, f_i(q)\, dq \right]$$

(by (5), (8), (9), and (20))

$$= \bar{n} \sum_{i=1}^{m} \left[\pi_i^*(y_i)\, \beta(y_i) - \int_{y_i}^{1} \pi_i^*(s)\, ds \right]$$

(by (17) and (27)).

The change of variables on the integration is $s = 1 - F_i(q)$. ∎

Proof of Theorem 9. The strategy of proof is to first show that (27) produces equal profits for any point on the border of the acceptance set, then produce a distribution of **y** over the acceptance set. As β is nonincreasing by (20), we have $0<\lambda<1/\beta(0)\leqslant 1/\beta(y)$ and can thus conclude that π^* is strictly decreasing. It is important to note that, by (1), (16), (17), and (18),

$$\beta(0)>1\geqslant\int_0^1 e^{\int_0^t [\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt. \tag{A9}$$

It is convenient to write (27) as

$$\pi^*(y)=\pi^*(0)\,e^{\int_0^y [\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}. \tag{A10}$$

Therefore, for **y** on the border of A^*,

$$\begin{aligned}
c/\bar{n}&=\sum_{i=1}^{m}\left[\pi^*(y_i)\,\beta(y_i)-\int_{y_i}^{1}\pi^*(s)\,ds\right]\\
&=\pi^*(0)\sum_{i=1}^{m}\left[\beta(y_i)\,e^{\int_0^{y_i}[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}-\int_{y_i}^{1}e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt\right]\\
&\qquad(\text{by (A10)})\\
&=\pi^*(0)\sum_{i=1}^{m}\left[\beta(0)-\int_0^1 e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt\right.\\
&\qquad\left.+\int_0^{y_i}e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\left(\beta'(t)+\beta(t)\frac{\lambda(1+\beta'(t))}{1-\lambda\beta(t)}+1\right)dt\right]\\
&=\pi^*(0)\left[m\left(\beta(0)-\int_0^1 e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt\right)\right.\\
&\qquad\left.+\sum_{i=1}^{m}\int_0^{y_i}e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,\frac{1+\beta'(t)}{1-\lambda\beta(t)}\,dt\right]\\
&=\pi^*(0)\left[m\left(\beta(0)-\int_0^1 e^{\int_0^t[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt\right)\right.\\
&\qquad\left.+\sum_{i=1}^{m}\frac{1}{\lambda}\left(e^{\int_0^{y_i}[\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}-1\right)\right]\\
&=m(c/\bar{n}-\lambda^{-1}\pi^*(0))+\lambda^{-1}\sum_{i=1}^{m}\pi^*(y_i).
\end{aligned}$$

The first equality follows from **y** on the border of the acceptance set and Lemma 8. The second substitutes (A10). The third equality follows from the Fundamental Theorem of Calculus, writing the first term in square brackets as the integral of its derivative, from 0 to y_i. The fourth obtains a common denominator for the terms in parentheses, the fifth integrates the last term, and the last equality uses (A10) and (27). If follows then that $\sum_{i=1}^{m} \pi_i^*(y_i)$ is constant for **y** on the border of A^*. Equations (19) and (26) yield

$$p_i = k_i + \frac{\bar{n}\pi^*(y_i)}{\varphi'(y_i)}, \tag{A11}$$

which is clearly decreasing in y_i, since π^* is decreasing and φ' is increasing.

There are various ways to specify a distribution on **y** so as to produce an equilibrium. The support of the distribution need not be the entire acceptance set; the only condition is that the marginal distributions on each y_i be uniform on [0, 1]. For concreteness, I will provide a distribution over the entire acceptance set. Begin with a uniform distribution on

$$S = \left\{ \mathbf{x} \in [0, 1]^m \,\middle|\, \sum_{i=1}^{m} x_i = 1 \right\}.$$

Then $y_i = (1 - x_i)^{m-1}$ is $\mathbf{U}[0, 1]$. Moreover, if $y_i = 0$, then $y_j = 1$ for all $j \neq i$. Thus,

$$\begin{aligned} c/\bar{n} &= \sum_{i=1}^{m} \pi^*(y_i)\, \beta(y_i) - \int_{y_i}^{1} \pi^*(s)\, ds && \text{(by Lemma 8)} \\ &= \pi^*(0)\, \beta(0) - \int_0^1 \pi^*(s)\, ds && \text{(setting } \mathbf{y} = (0, 1, ..., 1)\text{)} \\ &= \pi^*(0) \left[\beta(0) - \int_0^1 e^{\int_0^t [\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\, ds}\, dt \right], && \text{(by (27))} \end{aligned}$$

which determines $\pi^*(0)$ consistent with (27).

For this construction, $A^* = \{\mathbf{y} \in [0, 1]^m \mid \sum_{i=1}^{m} y_i^{1/(m-1)} = m - 1\}$. The actual distribution of prices is found by mapping the uniform distribution on S into the distribution on A^*, which, by (A11), produces the distribution of prices.

Total profits are

$$\begin{aligned} \sum_{i=1}^{m} \pi^*(y_i) &= \pi^*(0) + (m-1)\, \pi^*(1) && \text{(setting } \mathbf{y} = (0, 1, ..., 1)\text{)} \\ &= \frac{c(1 + (m-1)I)}{\bar{n}(\beta(0) - I)}, && \text{(by (A10))}, \end{aligned}$$

where

$$I=\int_0^1 e^{\int_0^t [\lambda(1+\beta'(s))/(1-\lambda\beta(s))]\,ds}\,dt<1, \text{ by (A9)}.$$

Profits are lower in this equilibrium than in any constant profits equilibrium (see (15)) if and only if, for $\mathbf{y}\in A^*$,

$$\sum_{i=1}^{m}\pi^*(y_i)<\frac{mc}{\bar{n}(\beta(0)-1)}$$

iff

$$\frac{1+(m-1)I}{\beta(0)-I}<\frac{m}{\beta(0)-1}$$

iff

$$(1+(m-1)I)(\beta(0)-1)<m(\beta(0)-I)$$

iff

$$((m-1)\,\beta(0)+1)(I-1)<0$$

which is true, by (A9). ∎

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